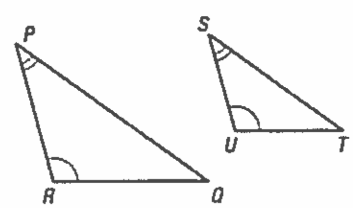
9.1 Similar Right Triangles

GOAL 1 PROPORTIONS IN RIGHT TRIANGLES

In Lesson 8.4, you learned that two triangles are similar if two of their corresponding angles are congruent. For example, $\triangle PQR \sim \triangle STU$. Recall that the corresponding side lengths of similar triangles are in proportion.

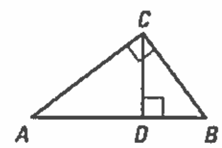
In the activity, you will see how a right triangle can be divided into two similar right triangles.



THEOREM

THEOREM 9.1

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.



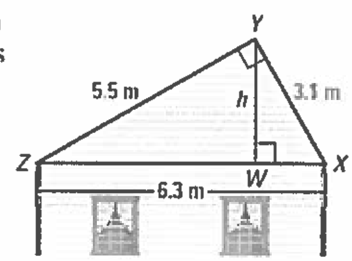
$\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, and $\triangle CBD \sim \triangle ACD$

EXAMPLE 1 ***Finding the Height of a Roof***

ROOF HEIGHT A roof has a cross section that is a right triangle. The diagram shows the approximate dimensions of this cross section.

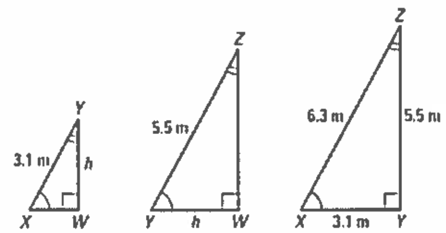
a. Identify the similar triangles.

b. Find the height h of the roof.



SOLUTION

a. You may find it helpful to sketch the three similar right triangles so that the corresponding angles and sides have the same orientation. Mark the congruent angles. Notice that some sides appear in more than one triangle. For instance, XY is the hypotenuse in $\triangle XYW$ and the shorter leg in $\triangle XZY$.



▶ $\triangle XYW \sim \triangle YZW \sim \triangle XZY$

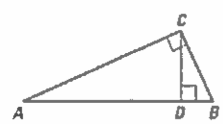
b. Use the fact that $\triangle XYW \sim \triangle XZY$ to write a proportion.

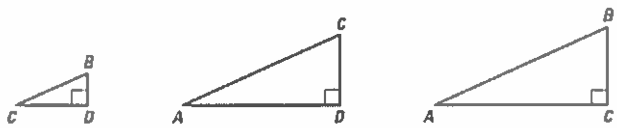
$\frac{YW}{ZY} = \frac{XY}{XZ}$	Corresponding side lengths are in proportion.
$\frac{h}{5.5} = \frac{3.1}{6.3}$	Substitute.
$6.3h = 5.5(3.1)$	Cross product property
$h \approx 2.7$	Solve for h.

▶ The height of the roof is about 2.7 meters.

GOAL 2 USING A GEOMETRIC MEAN TO SOLVE PROBLEMS

In right $\triangle ABC$, altitude $\overline{CD}$ is drawn to the hypotenuse, forming two smaller right triangles that are similar to $\triangle ABC$. From Theorem 9.1, you know that $\triangle CBD \sim \triangle ACD \sim \triangle ABC$.





Notice that $\overline{CD}$ is the longer leg of $\triangle CBD$ and the shorter leg of $\triangle ACD$. When you write a proportion comparing the leg lengths of $\triangle CBD$ and $\triangle ACD$, you can see that CD is the *geometric mean* of BD and AD.

$$\frac{\text{shorter leg of } \triangle CBD}{\text{shorter leg of } \triangle ACD} \quad \frac{BD}{CD} = \frac{CD}{AD} \quad \frac{\text{longer leg of } \triangle CBD}{\text{longer leg of } \triangle ACD}$$

Sides $\overline{CB}$ and $\overline{AC}$ also appear in more than one triangle. Their side lengths are also geometric means, as shown by the proportions below:

$$\frac{\text{hypotenuse of } \triangle ABC}{\text{hypotenuse of } \triangle CBD} \quad \frac{AB}{CB} = \frac{CB}{DB} \quad \frac{\text{shorter leg of } \triangle ABC}{\text{shorter leg of } \triangle CBD}$$

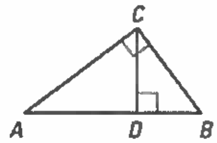
$$\frac{\text{hypotenuse of } \triangle ABC}{\text{hypotenuse of } \triangle ACD} \quad \frac{AB}{AC} = \frac{AC}{AD} \quad \frac{\text{longer leg of } \triangle ABC}{\text{longer leg of } \triangle ACD}$$

GEOMETRIC MEAN THEOREMS

THEOREM 9.2

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the geometric mean of the lengths of the two segments.



$$\frac{BD}{CD} = \frac{CD}{AD}$$

THEOREM 9.3

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

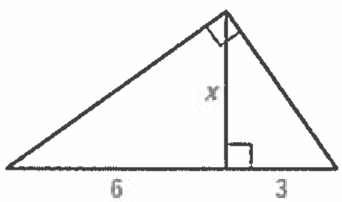
$$\frac{AB}{CB} = \frac{CB}{DB}$$

$$\frac{AB}{AC} = \frac{AC}{AD}$$

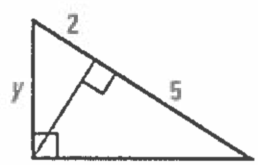
EXAMPLE 2 *Using a Geometric Mean*

Find the value of each variable.

a.



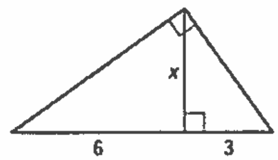
b.



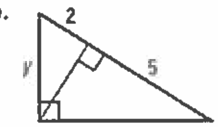
EXAMPLE 2 ***Using a Geometric Mean***

Find the value of each variable.

a.



b.



SOLUTION

a. Apply Theorem 9.2.

$$\frac{6}{x} = \frac{x}{3}$$

$$18 = x^2$$

$$\sqrt{18} = x$$

$$\sqrt{9} \cdot \sqrt{2} = x$$

$$3\sqrt{2} = x$$

b. Apply Theorem 9.3.

$$\frac{5+2}{y} = \frac{y}{2}$$

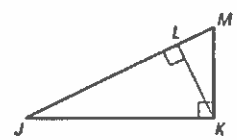
$$\frac{7}{y} = \frac{y}{2}$$

$$14 = y^2$$

$$\sqrt{14} = y$$

GUIDED PRACTICE

Vocabulary Check ✓ In Exercises 1–3, use the diagram at the right.



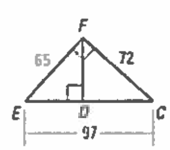
1. In the diagram, KL is the ___?___ of ML and JL.

Concept Check ✓

2. Complete the following statement: $\triangle JKL \sim \triangle$ ___?___ $\sim \triangle$ ___?___.
3. Which segment's length is the geometric mean of ML and MJ?

Skill Check ✓ In Exercises 4–9, use the diagram above. Complete the proportion.

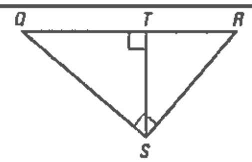
4. $\frac{KM}{KL} = \frac{?}{JK}$
5. $\frac{JM}{?} = \frac{JK}{JL}$
6. $\frac{?}{LK} = \frac{LK}{LM}$
7. $\frac{JM}{?} = \frac{KM}{LM}$
8. $\frac{LK}{LM} = \frac{JK}{?}$
9. $\frac{?}{JK} = \frac{MK}{MJ}$
10. Use the diagram at the right. Find DC. Then find DF. Round decimals to the nearest tenth.



SIMILAR TRIANGLES Use the diagram.

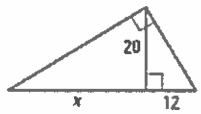
11. Sketch the three similar triangles in the diagram. Label the vertices.

12. Write similarity statements for the three triangles.

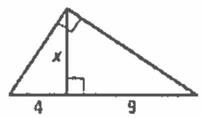


USING PROPORTIONS Complete and solve the proportion.

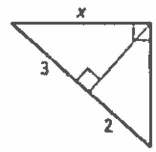
13. $\frac{x}{20} = \frac{?}{12}$



14. $\frac{4}{x} = \frac{x}{?}$

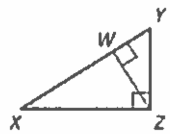


15. $\frac{5}{x} = \frac{x}{?}$

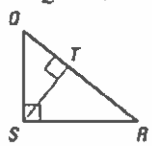


COMPLETING PROPORTIONS Write similarity statements for the three similar triangles in the diagram. Then complete the proportion.

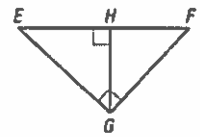
16. $\frac{XW}{ZW} = \frac{?}{YW}$



17. $\frac{QT}{SQ} = \frac{SQ}{?}$

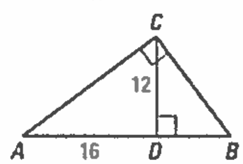


18. $\frac{?}{EG} = \frac{EG}{EF}$

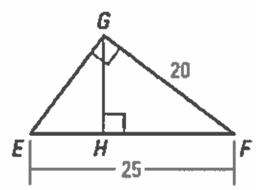


FINDING LENGTHS Write similarity statements for three triangles in the diagram. Then find the given length. Round decimals to the nearest tenth.

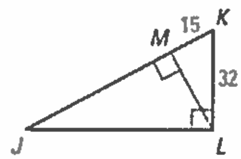
19. Find DB.



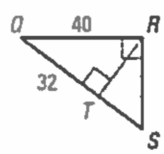
20. Find HF.



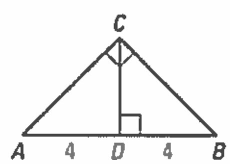
21. Find JK.



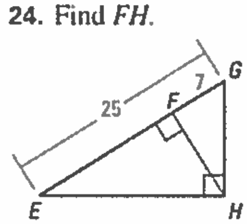
22. Find QS.



23. Find CD.

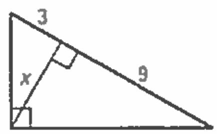


24. Find FH.



USING ALGEBRA Find the value of each variable.

25.



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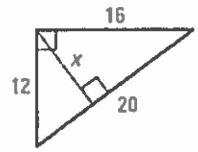

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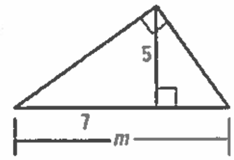

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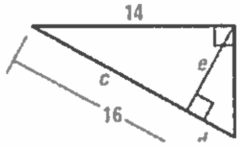

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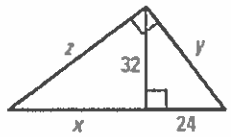

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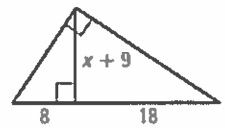
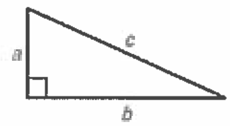


9.2 The Pythagorean Theorem

THEOREM

THEOREM 9.4 *Pythagorean Theorem*

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

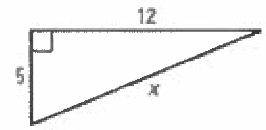


$c^2 = a^2 + b^2$

A **Pythagorean triple** is a set of three positive integers a, b, and c that satisfy the equation $c^2 = a^2 + b^2$. For example, the integers 3, 4, and 5 form a Pythagorean triple because $5^2 = 3^2 + 4^2$.

EXAMPLE 1 ***Finding the Length of a Hypotenuse***

Find the length of the hypotenuse of the right triangle. Tell whether the side lengths form a Pythagorean triple.



SOLUTION

EXAMPLE 1 ***Finding the Length of a Hypotenuse***

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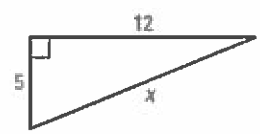


SOLUTION

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$ Pythagorean Theorem

$x^2 = 5^2 + 12^2$ Substitute.

$x^2 = 25 + 144$ Multiply.

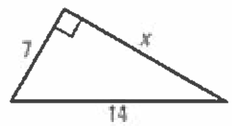
$x^2 = 169$ Add.

$x = 13$ Find the positive square root.

▶ Because the side lengths 5, 12, and 13 are integers, they form a Pythagorean triple.

EXAMPLE 2 ***Finding the Length of a Leg***

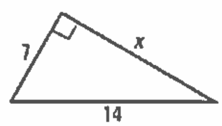
Find the length of the leg of the right triangle.



SOLUTION

EXAMPLE 2 ***Finding the Length of a Leg***

Find the length of the leg of the right triangle.

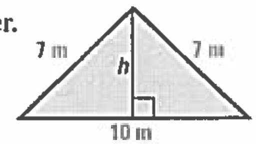


SOLUTION

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$	**Pythagorean Theorem**
$14^2 = 7^2 + x^2$	**Substitute.**
$196 = 49 + x^2$	**Multiply.**
$147 = x^2$	**Subtract 49 from each side.**
$\sqrt{147} = x$	**Find the positive square root.**
$\sqrt{49} \cdot \sqrt{3} = x$	**Use product property.**
$7\sqrt{3} = x$	**Simplify the radical.**

EXAMPLE 3 *Finding the Area of a Triangle*

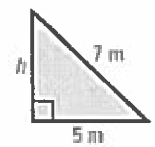
Find the area of the triangle to the nearest tenth of a meter.



SOLUTION

You are given that the base of the triangle is 10 meters, but you do not know the height h.

Because the triangle is isosceles, it can be divided into two congruent right triangles with the given dimensions. Use the Pythagorean Theorem to find the value of h.



$7^2 = 5^2 + h^2$	Pythagorean Theorem
$49 = 25 + h^2$	Multiply.
$24 = h^2$	Subtract 25 from both sides.
$\sqrt{24} = h$	Find the positive square root.

FINDING SIDE LENGTHS **Find the unknown side length. Simplify answers that are radicals. Tell whether the side lengths form a Pythagorean triple.**

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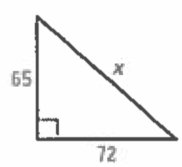

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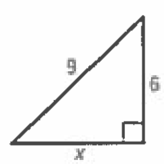

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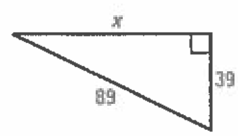

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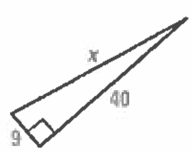

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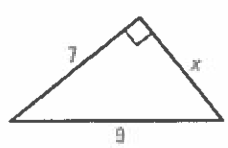

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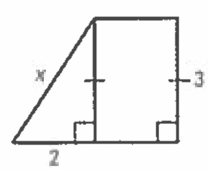

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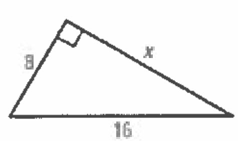

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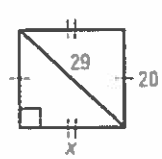

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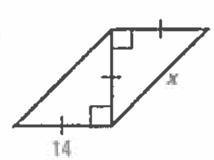
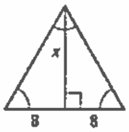


FINDING LENGTHS Find the value of x. Simplify answers that are radicals.

16.



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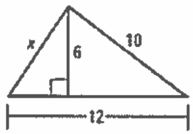

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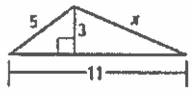
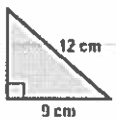


PYTHAGOREAN TRIPLES The variables r and s represent the lengths of the legs of a right triangle, and t represents the length of the hypotenuse. The values of r, s, and t form a Pythagorean triple. Find the unknown value.

19. $r = 12, s = 16$
20. $r = 9, s = 12$
21. $r = 18, t = 30$
22. $s = 20, t = 101$
23. $r = 35, t = 37$
24. $t = 757, s = 595$

FINDING AREA Find the area of the figure. Round decimal answers to the nearest tenth.

25.



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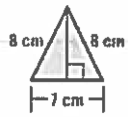
27.



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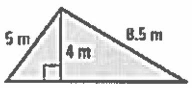

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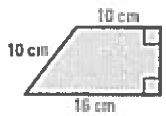

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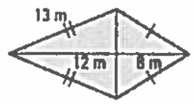
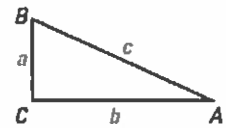


9.3 The Converse of the Pythagorean Theorem

THEOREM

THEOREM 9.5 *Converse of the Pythagorean Theorem*

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.

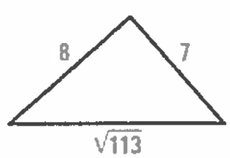


If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a right triangle.

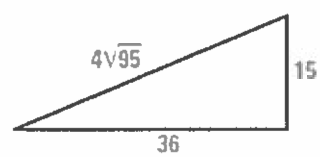
EXAMPLE 1 *Verifying Right Triangles*

The triangles below appear to be right triangles. Tell whether they are right triangles.

a.



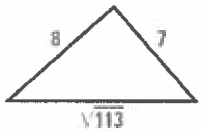
b.



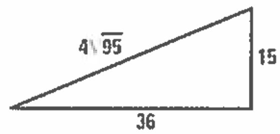
EXAMPLE 1 ***Verifying Right Triangles***

The triangles below appear to be right triangles. Tell whether they are right triangles.

a.



b.



SOLUTION

Let c represent the length of the longest side of the triangle. Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.

a. $(\sqrt{113})^2 \stackrel{?}{=} 7^2 + 8^2$

$113 \stackrel{?}{=} 49 + 64$

$113 = 113$ ✓

The triangle is a right triangle.

b. $(4\sqrt{95})^2 \stackrel{?}{=} 15^2 + 36^2$

$4^2 \cdot (\sqrt{95})^2 \stackrel{?}{=} 15^2 + 36^2$

$16 \cdot 95 \stackrel{?}{=} 225 + 1296$

$1520 \neq 1521$

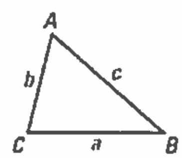
The triangle is not a right triangle.

THEOREMS

THEOREM 9.6

If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle is acute.

If $c^2 < a^2 + b^2$, then $\triangle ABC$ is acute.

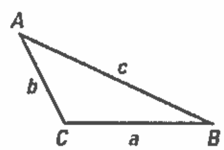


$c^2 < a^2 + b^2$

THEOREM 9.7

If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle is obtuse.

If $c^2 > a^2 + b^2$, then $\triangle ABC$ is obtuse.



$c^2 > a^2 + b^2$

EXAMPLE 2 ***Classifying Triangles***

Decide whether the set of numbers can represent the side lengths of a triangle. If they can, classify the triangle as *right*, *acute*, or *obtuse*.

a. 38, 77, 86

b. 10.5, 36.5, 37.5

SOLUTION

You can use the Triangle Inequality to confirm that each set of numbers can represent the side lengths of a triangle.

Compare the square of the length of the longest side with the sum of the squares of the lengths of the two shorter sides.

a.

$c^2 \underline{\ ?\ } a^2 + b^2$	Compare c^2 with $a^2 + b^2$.
$86^2 \underline{\ ?\ } 38^2 + 77^2$	Substitute.
$7396 \underline{\ ?\ } 1444 + 5929$	Multiply.
$7396 > 7373$	c^2 is greater than $a^2 + b^2$.

▶ Because $c^2 > a^2 + b^2$, the triangle is obtuse.

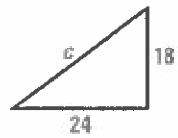
b.

$c^2 \underline{\ ?\ } a^2 + b^2$	Compare c^2 with $a^2 + b^2$.
$37.5^2 \underline{\ ?\ } 10.5^2 + 36.5^2$	Substitute.
$1406.25 \underline{\ ?\ } 110.25 + 1332.25$	Multiply.
$1406.25 < 1442.5$	c^2 is less than $a^2 + b^2$.

▶ Because $c^2 < a^2 + b^2$, the triangle is acute.

1. State the Converse of the Pythagorean Theorem in your own words.

2. Use the triangle shown at the right. Find values for c so that the triangle is acute, right, and obtuse.



In Exercises 3–6, match the side lengths with the appropriate description.

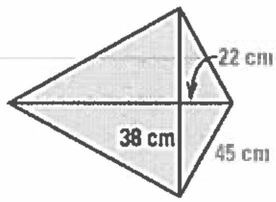
3. 2, 10, 11 — **A.** right triangle

4. 13, 5, 7 — **B.** acute triangle

5. 5, 11, 6 — **C.** obtuse triangle

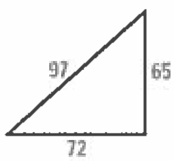
6. 6, 8, 10 — **D.** not a triangle

7. KITE DESIGN You are making the diamond-shaped kite shown at the right. You measure the crossbars to determine whether they are perpendicular. Are they? Explain.



VERIFYING RIGHT TRIANGLES **Tell whether the triangle is a right triangle.**

8.



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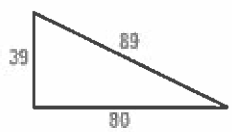

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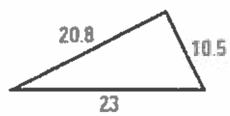

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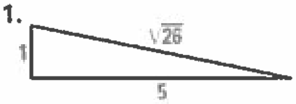

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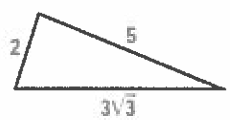

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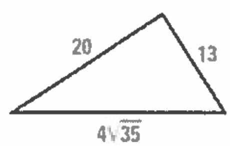
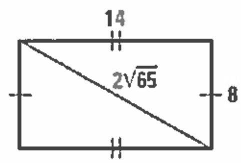


CLASSIFYING TRIANGLES Decide whether the numbers can represent the side lengths of a triangle. If they can, classify the triangle as *right, acute,* or *obtuse*.

14. 20, 99, 101
15. 21, 28, 35
16. 26, 10, 17
17. 2, 10, 12
18. 4, $\sqrt{67}$, 9
19. $\sqrt{13}$, 6, 7
20. 16, 30, 34
21. 10, 11, 14
22. 4, 5, 5
23. 17, 144, 145
24. 10, 49, 50
25. $\sqrt{5}$, 5, 5.5

CLASSIFYING QUADRILATERALS Classify the quadrilateral. Explain how you can prove that the quadrilateral is that type.

26.



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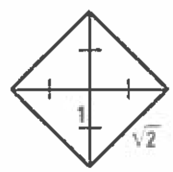

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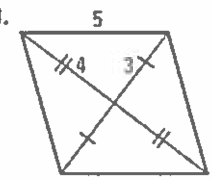


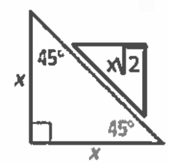
9.4 Special Right Triangles

Right triangles whose angle measures are 45°-45°-90° or 30°-60°-90° are called **special right triangles.** In the Activity on page 550, you may have noticed certain relationships among the side lengths of each of these special right triangles. The theorems below describe these relationships. Exercises 35 and 36 ask you to prove the theorems.

THEOREMS ABOUT SPECIAL RIGHT TRIANGLES

THEOREM 9.8 ***45°-45°-90° Triangle Theorem***

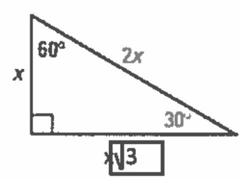
In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.



Hypotenuse = $\sqrt{2}$ · leg

THEOREM 9.9 ***30°-60°-90° Triangle Theorem***

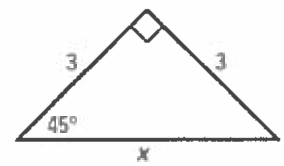
In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.



Hypotenuse = 2 · shorter leg
Longer leg = $\sqrt{3}$ · shorter leg

EXAMPLE 1 *Finding the Hypotenuse in a 45°-45°-90° Triangle*

Find the value of x.



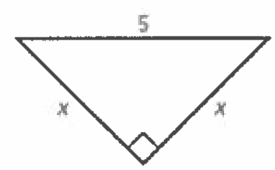
SOLUTION

By the Triangle Sum Theorem, the measure of the third angle is 45°. The triangle is a 45°-45°-90° right triangle, so the length x of the hypotenuse is $\sqrt{2}$ times the length of a leg.

$\text{Hypotenuse} = \sqrt{2} \cdot \text{leg}$	45°-45°-90° Triangle Theorem
$x = \sqrt{2} \cdot 3$	Substitute.
$x = 3\sqrt{2}$	Simplify.

EXAMPLE 2 *Finding a Leg in a 45°-45°-90° Triangle*

Find the value of x.

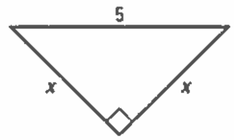


SOLUTION

Because the triangle is an isosceles right triangle, its base angles are congruent. The triangle is a 45°-45°-90° right triangle, so the length of the hypotenuse is $\sqrt{2}$ times the length x of a leg.

EXAMPLE 2 *Finding a Leg in a 45°-45°-90° Triangle*

Find the value of x.



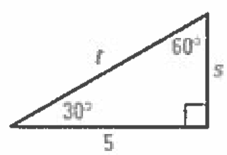
SOLUTION

Because the triangle is an isosceles right triangle, its base angles are congruent. The triangle is a 45°-45°-90° right triangle, so the length of the hypotenuse is $\sqrt{2}$ times the length x of a leg.

$\text{Hypotenuse} = \sqrt{2} \cdot \text{leg}$	45°-45°-90° Triangle Theorem
$5 = \sqrt{2} \cdot x$	Substitute.
$\frac{5}{\sqrt{2}} = \frac{\sqrt{2}x}{\sqrt{2}}$	Divide each side by $\sqrt{2}$.
$\frac{5}{\sqrt{2}} = x$	Simplify.
$\frac{\sqrt{2}}{\sqrt{2}} \cdot \frac{5}{\sqrt{2}} = x$	Multiply numerator and denominator by $\sqrt{2}$.
$\frac{5\sqrt{2}}{2} = x$	Simplify.

EXAMPLE 3 *Side Lengths in a 30°-60°-90° Triangle*

Find the values of s and t.

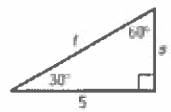


SOLUTION

Because the triangle is a 30°-60°-90° triangle, the longer leg is $\sqrt{3}$ times the length s of the shorter leg.

EXAMPLE 3 ***Side Lengths in a 30°-60°-90° Triangle***

Find the values of s and t.



SOLUTION

Because the triangle is a 30°-60°-90° triangle, the longer leg is $\sqrt{3}$ times the length s of the shorter leg.

Longer leg $= \sqrt{3} \cdot$ shorter leg	**30°-60°-90° Triangle Theorem**
$5 = \sqrt{3} \cdot s$	**Substitute.**
$\frac{5}{\sqrt{3}} = \frac{\sqrt{3} \cdot s}{\sqrt{3}}$	**Divide each side by $\sqrt{3}$.**
$\frac{5}{\sqrt{3}} = s$	**Simplify.**
$\frac{\sqrt{3}}{\sqrt{3}} \cdot \frac{5}{\sqrt{3}} = s$	**Multiply numerator and denominator by $\sqrt{3}$.**
$\frac{5\sqrt{3}}{3} = s$	**Simplify.**

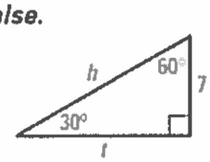
The length t of the hypotenuse is twice the length s of the shorter leg.

Hypotenuse $= 2 \cdot$ shorter leg	**30°-60°-90° Triangle Theorem**
$t = 2 \cdot \frac{5\sqrt{3}}{3}$	**Substitute.**
$t = \frac{10\sqrt{3}}{3}$	**Simplify.**

1. What is meant by the term *special right triangles*?

2. **CRITICAL THINKING** Explain why any two 30°-60°-90° triangles are similar.

Use the diagram to tell whether the equation is *true* or *false*.



3. $t = 7\sqrt{3}$

4. $t = \sqrt{3h}$

5. $h = 2t$

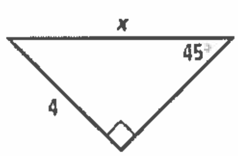
6. $h = 14$

7. $7 = \frac{h}{2}$

8. $7 = \frac{t}{\sqrt{3}}$

Find the value of each variable. Write answers in simplest radical form.

9.



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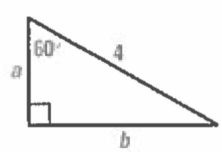

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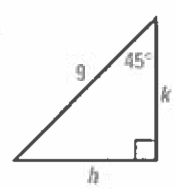
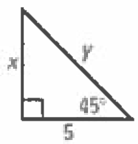
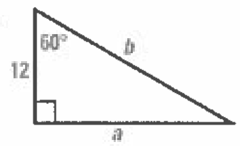
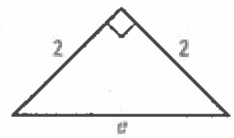
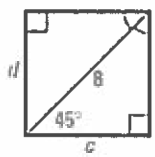


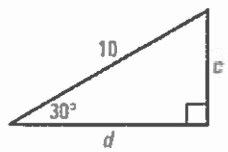
USING ALGEBRA Find the value of each variable. Write answers in simplest radical form.

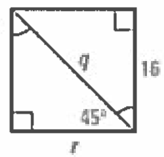
12. 

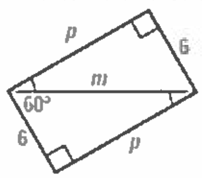
13. 

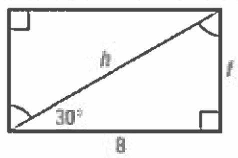
14. 

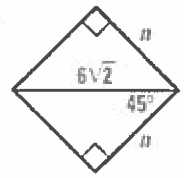
15. 

16. 

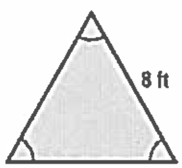
17. 

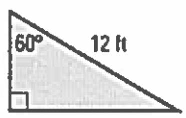
18. 

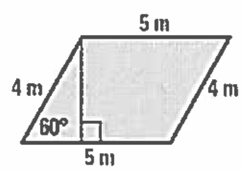
19. 

20. 

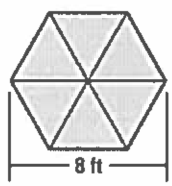
FINDING AREA Find the area of the figure. Round decimal answers to the nearest tenth.

24. 

25. 

26. 

27. **AREA OF A WINDOW** A hexagonal window consists of six congruent panes of glass. Each pane is an equilateral triangle. Find the area of the entire window.



9.5 Trigonometric Ratios

GOAL 1 FINDING TRIGONOMETRIC RATIOS

A **trigonometric ratio** is a ratio of the lengths of two sides of a right triangle. The word *trigonometry* is derived from the ancient Greek language and means measurement of triangles. The three basic trigonometric ratios are **sine**, **cosine**, and **tangent**, which are abbreviated as *sin*, *cos*, and *tan*, respectively.

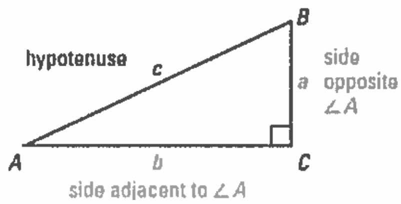
TRIGONOMETRIC RATIOS

Let $\triangle ABC$ be a right triangle. The sine, the cosine, and the tangent of the acute angle $\angle A$ are defined as follows.

$$\sin A = \frac{\text{side opposite } \angle A}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan A = \frac{\text{side opposite } \angle A}{\text{side adjacent to } \angle A} = \frac{a}{b}$$

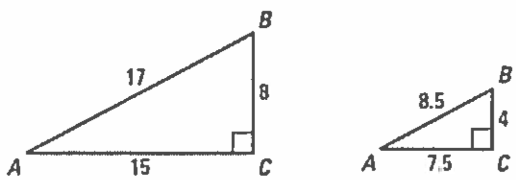


EXAMPLE 1 ***Finding Trigonometric Ratios***

Compare the sine, the cosine, and the tangent ratios for $\angle A$ in each triangle below.

SOLUTION

By the SSS Similarity Theorem, the triangles are similar. Their corresponding sides are in proportion, which implies that the trigonometric ratios for $\angle A$ in each triangle are the same.

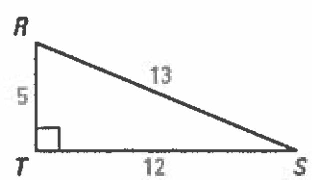


	Large triangle	Small triangle
$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$	$\frac{8}{17} \approx 0.4706$	$\frac{4}{8.5} \approx 0.4706$
$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$	$\frac{15}{17} \approx 0.8824$	$\frac{7.5}{8.5} \approx 0.8824$
$\tan A = \frac{\text{opposite}}{\text{adjacent}}$	$\frac{8}{15} \approx 0.5333$	$\frac{4}{7.5} \approx 0.5333$

EXAMPLE 2 ***Finding Trigonometric Ratios***

Find the sine, the cosine, and the tangent of the indicated angle.

a. $\angle S$ **b.** $\angle R$



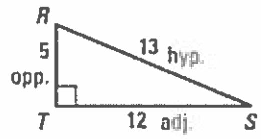
SOLUTION

a. The length of the hypotenuse is 13. For $\angle S$, the length of the opposite side is 5, and the length of the adjacent side is 12.

$$\sin S = \frac{\text{opp.}}{\text{hyp.}} = \frac{5}{13} \approx 0.3846$$

$$\cos S = \frac{\text{adj.}}{\text{hyp.}} = \frac{12}{13} \approx 0.9231$$

$$\tan S = \frac{\text{opp.}}{\text{adj.}} = \frac{5}{12} \approx 0.4167$$

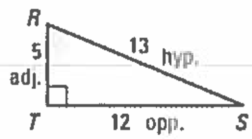


b. The length of the hypotenuse is 13. For $\angle R$, the length of the opposite side is 12, and the length of the adjacent side is 5.

$$\sin R = \frac{\text{opp.}}{\text{hyp.}} = \frac{12}{13} \approx 0.9231$$

$$\cos R = \frac{\text{adj.}}{\text{hyp.}} = \frac{5}{13} \approx 0.3846$$

$$\tan R = \frac{\text{opp.}}{\text{adj.}} = \frac{12}{5} = 2.4$$

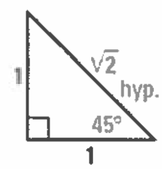


EXAMPLE 3 ***Trigonometric Ratios for 45°***

Find the sine, the cosine, and the tangent of 45°.

SOLUTION

Begin by sketching a 45°-45°-90° triangle. Because all such triangles are similar, you can make calculations simple by choosing 1 as the length of each leg. From Theorem 9.8 on page 551, it follows that the length of the hypotenuse is $\sqrt{2}$.

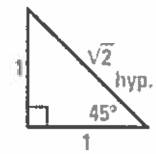


EXAMPLE 3 *Trigonometric Ratios for 45°*

Find the sine, the cosine, and the tangent of 45°.

SOLUTION

Begin by sketching a 45°-45°-90° triangle. Because all such triangles are similar, you can make calculations simple by choosing 1 as the length of each leg. From Theorem 9.8 on page 551, it follows that the length of the hypotenuse is $\sqrt{2}$.



$$\sin 45° = \frac{\text{opp.}}{\text{hyp.}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \approx 0.7071$$

$$\cos 45° = \frac{\text{adj.}}{\text{hyp.}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \approx 0.7071$$

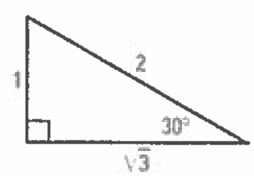
$$\tan 45° = \frac{\text{opp.}}{\text{adj.}} = \frac{1}{1} = 1$$

EXAMPLE 4 *Trigonometric Ratios for 30°*

Find the sine, the cosine, and the tangent of 30°.

SOLUTION

Begin by sketching a 30°-60°-90° triangle. To make the calculations simple, you can choose 1 as the length of the shorter leg. From Theorem 9.9 on page 551, it follows that the length of the longer leg is $\sqrt{3}$ and the length of the hypotenuse is 2.

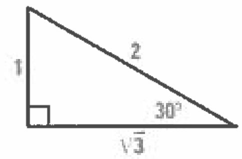


EXAMPLE 4 *Trigonometric Ratios for 30°*

Find the sine, the cosine, and the tangent of 30°.

SOLUTION

Begin by sketching a 30°-60°-90° triangle. To make the calculations simple, you can choose 1 as the length of the shorter leg. From Theorem 9.9 on page 551, it follows that the length of the longer leg is $\sqrt{3}$ and the length of the hypotenuse is 2.



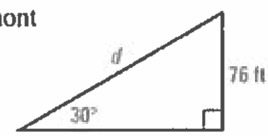
$$\sin 30° = \frac{\text{opp.}}{\text{hyp.}} = \frac{1}{2} = 0.5$$

$$\cos 30° = \frac{\text{adj.}}{\text{hyp.}} = \frac{\sqrt{3}}{2} \approx 0.8660$$

$$\tan 30° = \frac{\text{opp.}}{\text{adj.}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \approx 0.5774$$

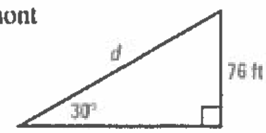
EXAMPLE 7 *Estimating a Distance*

ESCALATORS The escalator at the Wilshire/Vermont Metro Rail Station in Los Angeles rises 76 feet at a 30° angle. To find the distance d a person travels on the escalator stairs, you can write a trigonometric ratio that involves the hypotenuse and the known leg length of 76 feet.



EXAMPLE 7 ***Estimating a Distance***

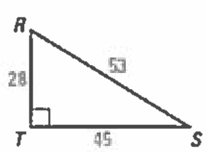
ESCALATORS The escalator at the Wilshire/Vermont Metro Rail Station in Los Angeles rises 76 feet at a 30° angle. To find the distance d a person travels on the escalator stairs, you can write a trigonometric ratio that involves the hypotenuse and the known leg length of 76 feet.



$\sin 30° = \frac{\text{opposite}}{\text{hypotenuse}}$	**Write ratio for sine of 30°.**
$\sin 30° = \frac{76}{d}$	**Substitute.**
$d \sin 30° = 76$	**Multiply each side by d.**
$d = \frac{76}{\sin 30°}$	**Divide each side by sin 30°.**
$d = \frac{76}{0.5}$	**Substitute 0.5 for sin 30°.**
$d = 152$	**Simplify.**

▶ A person travels 152 feet on the escalator stairs.

FINDING TRIGONOMETRIC RATIOS Find the sine, the cosine, and the tangent of the acute angles of the triangle. Express each value as a decimal rounded to four places.

10.


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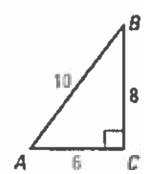

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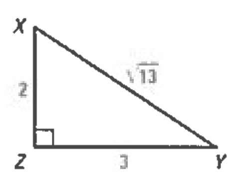

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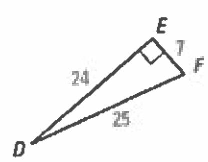

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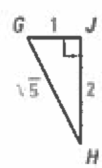

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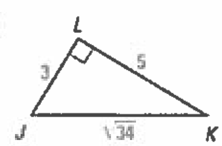


CALCULATOR Use a calculator to approximate the given value to four decimal places.

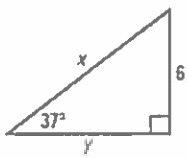
16. sin 48° **17.** cos 13° **18.** tan 81° **19.** sin 27°

20. cos 70° **21.** tan 2° **22.** sin 78° **23.** cos 36°

24. tan 23° **25.** cos 63° **26.** sin 56° **27.** tan 66°

USING TRIGONOMETRIC RATIOS **Find the value of each variable. Round decimals to the nearest tenth.**

28.



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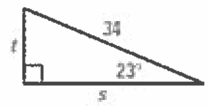

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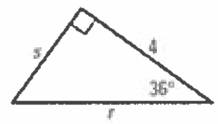

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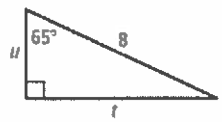

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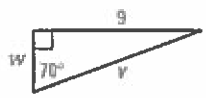

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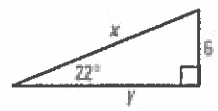
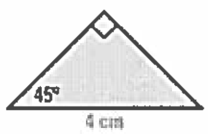
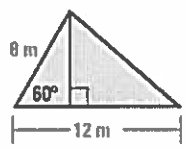


FINDING AREA **Find the area of the triangle. Round decimals to the nearest tenth.**

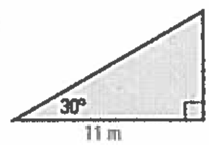
34.



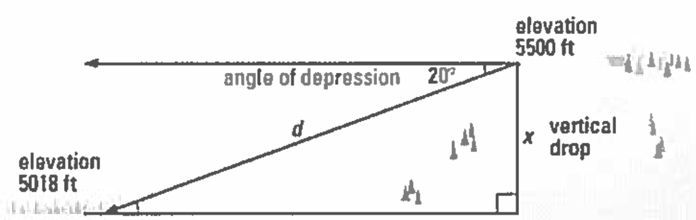
35.



36.



39. **SKI SLOPE** Suppose you stand at the top of a ski slope and look down at the bottom. The angle that your line of sight makes with a line drawn horizontally is called the *angle of depression*, as shown below. The *vertical drop* is the difference in the elevations of the top and the bottom of the slope. Find the vertical drop x of the slope in the diagram. Then estimate the distance d a person skiing would travel on this slope.



9.6 Solving Right Triangles

GOAL 1 SOLVING A RIGHT TRIANGLE

Every right triangle has one right angle, two acute angles, one hypotenuse, and two legs. To **solve a right triangle** means to determine the measures of all six parts. You can solve a right triangle if you know either of the following:

- Two side lengths
- One side length and one acute angle measure

As you learned in Lesson 9.5, you can use the side lengths of a right triangle to find trigonometric ratios for the acute angles of the triangle. As you will see in this lesson, once you know the sine, the cosine, or the tangent of an acute angle, you can use a calculator to find the measure of the angle.

In general, for an acute angle A:

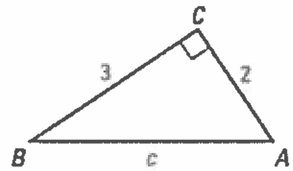
if $\sin A = x$, then $\sin^{-1} x = m\angle A$. ← The expression $\sin^{-1} x$ is read as "the inverse sine of x."

if $\cos A = y$, then $\cos^{-1} y = m\angle A$.

if $\tan A = z$, then $\tan^{-1} z = m\angle A$.

EXAMPLE 1 ***Solving a Right Triangle***

Solve the right triangle. Round decimals to the nearest tenth.



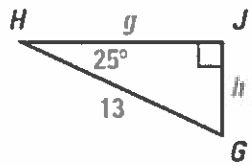
SOLUTION

Begin by using the Pythagorean Theorem to find the length of the hypoter

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$	Pythagorean Theorem
$c^2 = 3^2 + 2^2$	Substitute.
$c^2 = 13$	Simplify.
$c = \sqrt{13}$	Find the positive square root.
$c \approx 3.6$	Use a calculator to approximate.

EXAMPLE 2 ***Solving a Right Triangle***

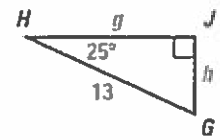
Solve the right triangle. Round decimals to the nearest tenth.



SOLUTION

EXAMPLE 2 Solving a Right Triangle

Solve the right triangle. Round decimals to the nearest tenth.



SOLUTION

Use trigonometric ratios to find the values of g and h.

$$\sin H = \frac{\text{opp.}}{\text{hyp.}} \qquad \cos H = \frac{\text{adj.}}{\text{hyp.}}$$

$$\sin 25° = \frac{h}{13} \qquad \cos 25° = \frac{g}{13}$$

$$13 \sin 25° = h \qquad 13 \cos 25° = g$$

$$13(0.4226) \approx h \qquad 13(0.9063) \approx g$$

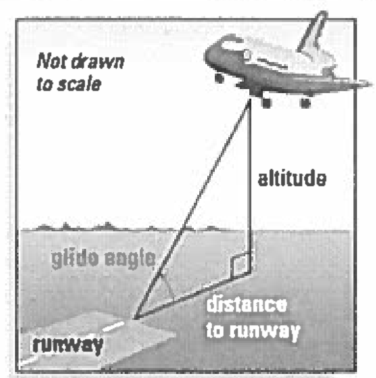
$$5.5 \approx h \qquad 11.8 \approx g$$

Because $\angle H$ and $\angle G$ are complements, you can write

$$m\angle G = 90° - m\angle H = 90° - 25° = 65°.$$

EXAMPLE 3 Solving a Right Triangle

SPACE SHUTTLE During its approach to Earth, the space shuttle's glide angle changes.

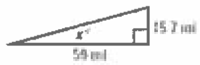


a. When the shuttle's altitude is about 15.7 miles, its horizontal distance to the runway is about 59 miles. What is its glide angle? Round your answer to the nearest tenth.

b. When the space shuttle is 5 miles from the runway, its glide angle is about 19°. Find the shuttle's altitude at this point in its descent. Round your answer to the nearest tenth.

SOLUTION

a. Sketch a right triangle to model the situation. Let $x° =$ the measure of the shuttle's glide angle. You can use the tangent ratio and a calculator to find the approximate value of x.



$\tan x° = \frac{\text{opp.}}{\text{adj.}}$

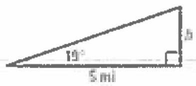
$\tan x° = \frac{15.7}{59}$ **Substitute.**

$x =$ (15.7 ÷ 59) 2nd TAN **Use a calculator to find $\tan^{-1}\left(\frac{15.7}{59}\right)$.**

$x \approx 14.9$

▶ When the space shuttle's altitude is about 15.7 miles, the glide angle is about 14.9°.

b. Sketch a right triangle to model the situation. Let $h =$ the altitude of the shuttle. You can use the tangent ratio and a calculator to find the approximate value of h.



$\tan 19° = \frac{\text{opp.}}{\text{adj.}}$

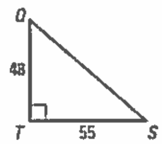
$\tan 19° = \frac{h}{5}$ **Substitute.**

$0.3443 \approx \frac{h}{5}$ **Use a calculator.**

$1.7 \approx h$ **Multiply each side by 5.**

▶ The shuttle's altitude is about 1.7 miles.

FINDING MEASUREMENTS Use the diagram to find the indicated measurement. Round your answer to the nearest tenth.



11. QS **12.** $m\angle Q$ **13.** $m\angle S$

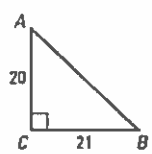
CALCULATOR In Exercises 14–21, $\angle A$ is an acute angle. Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.

14. $\tan A = 0.5$ **15.** $\tan A = 1.0$ **16.** $\sin A = 0.5$ **17.** $\sin A = 0.35$

18. $\cos A = 0.15$ **19.** $\cos A = 0.64$ **20.** $\tan A = 2.2$ **21.** $\sin A = 0.11$

SOLVING RIGHT TRIANGLES Solve the right triangle. Round decimals to the nearest tenth.

22.



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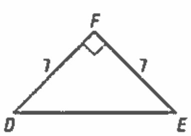

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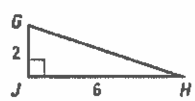

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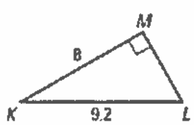

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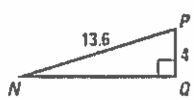

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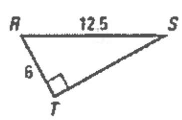
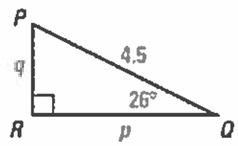


SOLVING RIGHT TRIANGLES Solve the right triangle. Round decimals to the nearest tenth.

28.



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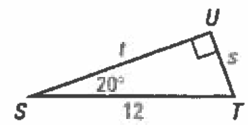

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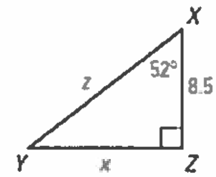

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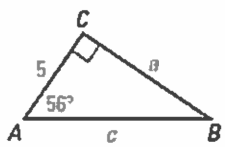

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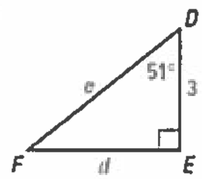

33.