# 8.1 Ratio and Proportion

If $a$ and $b$ are two quantities that are measured in the *same* units, then the **ratio of $a$ to $b$** is $\frac{a}{b}$. The ratio of $a$ to $b$ can also be written as $a:b$. Because a ratio is a quotient, its denominator cannot be *zero*.

Ratios are usually expressed in simplified form. For instance, the ratio of 6:8 is usually simplified as 3:4.

**EXAMPLE 1** *Simplifying Ratios*

Simplify the ratios.

a. $\frac{12 \text{ cm}}{4 \text{ m}}$

b. $\frac{6 \text{ ft}}{18 \text{ in.}}$

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**SOLUTION**

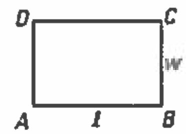
To simplify ratios with unlike units, convert to like units so that the units divide out. Then simplify the fraction, if possible.

a. $\frac{12 \text{ cm}}{4 \text{ m}} = \frac{12 \text{ cm}}{4 \cdot 100 \text{ cm}} = \frac{12}{400} = \frac{3}{100}$

b. $\frac{6 \text{ ft}}{18 \text{ in.}} = \frac{6 \cdot 12 \text{ in.}}{18 \text{ in.}} = \frac{72}{18} = \frac{4}{1}$

## EXAMPLE 2 Using Ratios

The perimeter of rectangle $ABCD$ is 60 centimeters. The ratio of $AB:BC$ is 3:2. Find the length and width of the rectangle.

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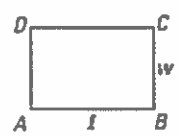



**SOLUTION**

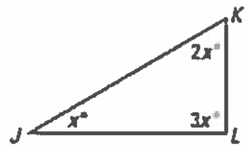
Because the ratio of $AB:BC$ is 3:2, you can represent the length $AB$ as $3x$ and the width $BC$ as $2x$.

| | |
|---|---|
| $2\ell + 2w = P$ | Formula for perimeter of rectangle |
| $2(3x) + 2(2x) = 60$ | Substitute for $\ell$, $w$, and $P$. |
| $6x + 4x = 60$ | Multiply. |
| $10x = 60$ | Combine like terms. |
| $x = 6$ | Divide each side by 10. |

▶ So, $ABCD$ has a length of 18 centimeters and a width of 12 centimeters.

**EXAMPLE 3** *Using Extended Ratios*

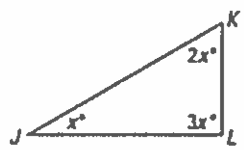
The measure of the angles in $\triangle JKL$ are in the *extended ratio* of $1:2:3$. Find the measures of the angles.




**SOLUTION**

**EXAMPLE 3** *Using Extended Ratios*

The measure of the angles in $\triangle JKL$ are in the *extended ratio* of $1:2:3$. Find the measures of the angles.




**SOLUTION**

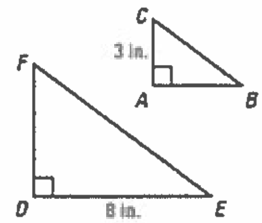
Begin by sketching a triangle. Then use the extended ratio of $1:2:3$ to label the measures of the angles as $x°$, $2x°$, and $3x°$.

| | |
|---|---|
| $x° + 2x° + 3x° = 180°$ | Triangle Sum Theorem |
| $6x = 180$ | Combine like terms. |
| $x = 30$ | Divide each side by 6. |

▶ So, the angle measures are $30°$, $2(30°) = 60°$, and $3(30°) = 90°$.

**EXAMPLE 4** *Using Ratios*

The ratios of the side lengths of $\triangle DEF$ to the corresponding side lengths of $\triangle ABC$ are 2 : 1. Find the unknown lengths.




**SOLUTION**

- $DE$ is twice $AB$ and $DE = 8$, so $AB = \frac{1}{2}(8) = 4$.
- Using the Pythagorean Theorem, you can determine that $BC = 5$.
- $DF$ is twice $AC$ and $AC = 3$, so $DF = 2(3) = 6$.
- $EF$ is twice $BC$ and $BC = 5$, so $EF = 2(5) = 10$.

**GOAL 2** **USING PROPORTIONS**

An equation that equates two ratios is a **proportion**. For instance, if the ratio $\frac{a}{b}$ is equal to the ratio $\frac{c}{d}$, then the following proportion can be written:

Means Extremes

$$\frac{a}{b} = \frac{c}{d}$$

The numbers $a$ and $d$ are the **extremes** of the proportion. The numbers $b$ and $c$ are the **means** of the proportion.

PROPERTIES OF PROPORTIONS

1. **CROSS PRODUCT PROPERTY** The product of the extremes equals the product of the means.

   If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

2. **RECIPROCAL PROPERTY** If two ratios are equal, then their reciprocals are also equal.

   If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$.

## EXAMPLE 5 *Solving Proportions*

Solve the proportions.

a. $\frac{4}{x} = \frac{5}{7}$

b. $\frac{3}{y+2} = \frac{2}{y}$

**EXAMPLE 5** ***Solving Proportions***

Solve the proportions.

**a.** $\frac{4}{x} = \frac{5}{7}$ **b.** $\frac{3}{y+2} = \frac{2}{y}$

**SOLUTION**

**a.** $\frac{4}{x} = \frac{5}{7}$ Write original proportion.

$\frac{x}{4} = \frac{7}{5}$ Reciprocal property

$x = 4\left(\frac{7}{5}\right)$ Multiply each side by 4.

$x = \frac{28}{5}$ Simplify.

**b.** $\frac{3}{y+2} = \frac{2}{y}$ Write original proportion.

$3y = 2(y + 2)$ Cross product property

$3y = 2y + 4$ Distributive property

$y = 4$ Subtract $2y$ from each side.

▶ The solution is 4. Check this by substituting in the original proportion.

## GUIDED PRACTICE

**Vocabulary Check**

1. In the proportion $\frac{r}{s} = \frac{p}{q}$, the variables $s$ and $p$ are the _?_ of the proportion and $r$ and $q$ are the _?_ of the proportion.

**Concept Check**

ERROR ANALYSIS **In Exercises 2 and 3, find and correct the errors.**

2. A table is 18 inches wide and 3 feet long. The ratio of length to width is 1 : 6.

3. $\frac{10}{x+6} = \frac{4}{x}$

$10x = 4x + 6$

$6x = 6$

$x = 1$

**Skill Check**

**Given that the track team won 8 meets and lost 2, find the ratios.**

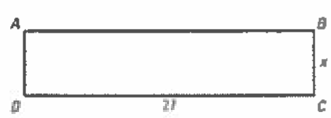
4. What is the ratio of wins to losses? What is the ratio of losses to wins?

5. What is the ratio of wins to the total number of track meets?

**In Exercises 6–8, solve the proportion.**

6. $\frac{2}{x} = \frac{3}{9}$ 7. $\frac{5}{8} = \frac{6}{z}$ 8. $\frac{2}{b+3} = \frac{4}{b}$

9. The ratio $BC:DC$ is $2:9$. Find the value of $x$.

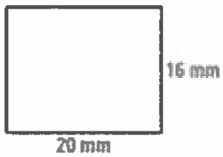


**SIMPLIFYING RATIOS Simplify the ratio.**

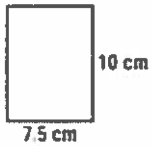
10. $\frac{16 \text{ students}}{24 \text{ students}}$ 11. $\frac{48 \text{ marbles}}{8 \text{ marbles}}$ 12. $\frac{22 \text{ feet}}{52 \text{ feet}}$ 13. $\frac{6 \text{ meters}}{9 \text{ meters}}$

**WRITING RATIOS Find the width to length ratio of each rectangle. Then simplify the ratio.**

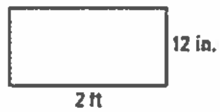
14.




15.




16.




**CONVERTING UNITS Rewrite the fraction so that the numerator and denominator have the same units. Then simplify.**

17. $\frac{3 \text{ ft}}{12 \text{ in.}}$ 18. $\frac{60 \text{ cm}}{1 \text{ m}}$ 19. $\frac{350 \text{ g}}{1 \text{ kg}}$ 20. $\frac{2 \text{ mi}}{3000 \text{ ft}}$

21. $\frac{6 \text{ yd}}{10 \text{ ft}}$ 22. $\frac{2 \text{ lb}}{20 \text{ oz}}$ 23. $\frac{400 \text{ m}}{0.5 \text{ km}}$ 24. $\frac{20 \text{ oz}}{4 \text{ lb}}$

**FINDING RATIOS Use the number line to find the ratio of the distances.**


25. $\frac{AB}{CD} = \underline{\ ?\ }$ 26. $\frac{BD}{CF} = \underline{\ ?\ }$ 27. $\frac{BF}{AD} = \underline{\ ?\ }$ 28. $\frac{CF}{AB} = \underline{\ ?\ }$

29. The perimeter of a rectangle is 84 feet. The ratio of the width to the length is 2 : 5. Find the length and the width.

30. The area of a rectangle is 108 cm$^2$. The ratio of the width to the length is 3 : 4. Find the length and the width.

31. The measures of the angles in a triangle are in the extended ratio of 1 : 4 : 7. Find the measures of the angles.

32. The measures of the angles in a triangle are in the extended ratio of 2 : 15 : 19. Find the measures of the angles.

**SOLVING PROPORTIONS** **Solve the proportion.**

33. $\frac{x}{4} = \frac{5}{7}$
34. $\frac{y}{8} = \frac{9}{10}$
35. $\frac{7}{z} = \frac{10}{25}$
36. $\frac{4}{b} = \frac{10}{3}$
37. $\frac{30}{5} = \frac{14}{c}$
38. $\frac{16}{3} = \frac{d}{6}$
39. $\frac{5}{x+3} = \frac{4}{x}$
40. $\frac{4}{y-3} = \frac{8}{y}$
41. $\frac{7}{2z+5} = \frac{3}{z}$
42. $\frac{3x-8}{6} = \frac{2x}{10}$
43. $\frac{5y-8}{7} = \frac{5y}{6}$
44. $\frac{4}{2z+6} = \frac{10}{7z-2}$

# 8.2 Problem Solving in Geometry with Proportions

**ADDITIONAL PROPERTIES OF PROPORTIONS**

3. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$.

4. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$.

## EXAMPLE 1 *Using Properties of Proportions*

Tell whether the statement is true.

**a.** If $\frac{p}{6} = \frac{r}{10}$, then $\frac{p}{r} = \frac{3}{5}$.

**b.** If $\frac{a}{3} = \frac{c}{4}$, then $\frac{a+3}{3} = \frac{c+3}{4}$.

**EXAMPLE 1** *Using Properties of Proportions*

Tell whether the statement is true.

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**SOLUTION**

a. $\frac{p}{6} = \frac{r}{10}$ Given

$\frac{p}{r} = \frac{6}{10}$ If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$.

$\frac{p}{r} = \frac{3}{5}$ Simplify.

▶ The statement is true.

b. $\frac{a}{3} = \frac{c}{4}$ Given

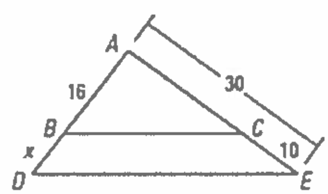
$\frac{a+3}{3} = \frac{c+4}{4}$ If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$.

Because $\frac{c+4}{4} \neq \frac{c+3}{4}$, the conclusions are not equivalent.

▶ The statement is false.

**EXAMPLE 2** *Using Properties of Proportions*

In the diagram $\frac{AB}{BD} = \frac{AC}{CE}$. Find the length of $\overline{BD}$.

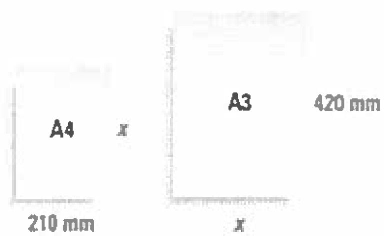


The geometric mean of two positive numbers $a$ and $b$ is the positive number $x$ such that $\frac{a}{x} = \frac{x}{b}$. If you solve this proportion for $x$, you find that $x = \sqrt{a \cdot b}$, which is a positive number.

For example, the geometric mean of 8 and 18 is 12, because $\frac{8}{12} = \frac{12}{18}$, and also because $\sqrt{8 \cdot 18} = \sqrt{144} = 12$.

**EXAMPLE 3** *Using a Geometric Mean*

**PAPER SIZES** International standard paper sizes are commonly used all over the world. The various sizes all have the same width-to-length ratios. Two sizes of paper are shown, called A4 and A3. The distance labeled $x$ is the geometric mean of 210 mm and 420 mm. Find the value of $x$.



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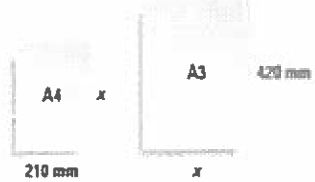
**PAPER SIZES** International standard paper sizes are commonly used all over the world. The various sizes all have the same width-to-length ratios. Two sizes of paper are shown, called A4 and A3. The distance labeled $x$ is the geometric mean of 210 mm and 420 mm. Find the value of $x$.




**SOLUTION**

| | |
|---|---|
| $\frac{210}{x} = \frac{x}{420}$ | Write proportion. |
| $x^2 = 210 \cdot 420$ | Cross product property |
| $x = \sqrt{210 \cdot 420}$ | Simplify. |
| $x = \sqrt{210 \cdot 210 \cdot 2}$ | Factor. |
| $x = 210\sqrt{2}$ | Simplify. |

▶ The geometric mean of 210 and 420 is $210\sqrt{2}$, or about 297. So, the distance labeled $x$ in the diagram is about 297 mm.

**LOGICAL REASONING** Complete the sentence.

9. If $\frac{2}{x} = \frac{7}{y}$, then $\frac{2}{7} = \frac{?}{?}$.

10. If $\frac{x}{6} = \frac{y}{34}$, then $\frac{x}{y} = \frac{?}{?}$.

11. If $\frac{x}{5} = \frac{y}{12}$, then $\frac{x+5}{5} = \frac{?}{?}$.

12. If $\frac{13}{7} = \frac{x}{y}$, then $\frac{20}{7} = \frac{?}{?}$.

**LOGICAL REASONING** Decide whether the statement is *true* or *false*.

13. If $\frac{7}{a} = \frac{b}{2}$, then $\frac{7+a}{a} = \frac{b+2}{2}$.

14. If $\frac{3}{4} = \frac{p}{r}$, then $\frac{4}{3} = \frac{p}{r}$.

15. If $\frac{c}{6} = \frac{d+2}{10}$, then $\frac{c}{d+2} = \frac{6}{10}$.

16. If $\frac{12+m}{12} = \frac{3+n}{n}$, then $\frac{m}{12} = \frac{3}{n}$.

**GEOMETRIC MEAN** Find the geometric mean of the two numbers.

17. 3 and 27

18. 4 and 16

19. 7 and 28

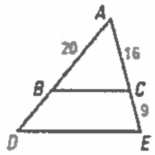
20. 2 and 40

21. 8 and 20

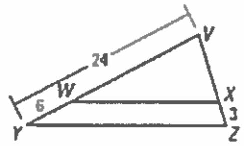
22. 5 and 15

**PROPERTIES OF PROPORTIONS** **Use the diagram and the given information to find the unknown length.**

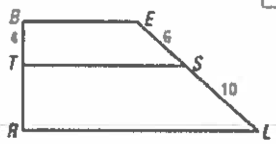
**23. GIVEN** ▶ $\frac{AB}{BD} = \frac{AC}{CE}$, find $BD$. ☐




**24. GIVEN** ▶ $\frac{VW}{WY} = \frac{VX}{XZ}$, find $VX$. ☐

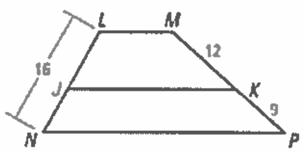



**25. GIVEN** ▶ $\frac{BT}{TR} = \frac{ES}{SL}$, find $TR$. ☐

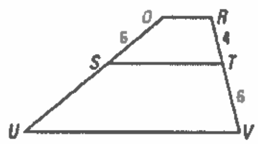



**26. GIVEN** ▶ $\frac{SP}{SK} = \frac{SQ}{SJ}$, find $SQ$. ☐


**27. GIVEN** ▶ $\frac{LJ}{JN} = \frac{MK}{KP}$, find $JN$. ☐




**28. GIVEN** ▶ $\frac{QU}{QS} = \frac{RV}{RT}$, find $SU$. ☐

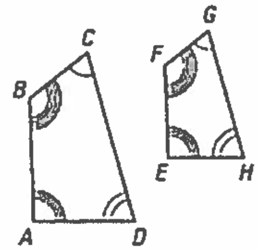



# 8.3 Similar Polygons

## GOAL 1 IDENTIFYING SIMILAR POLYGONS

When there is a correspondence between two polygons such that their corresponding angles are congruent and the lengths of corresponding sides are proportional the two polygons are called **similar polygons.**

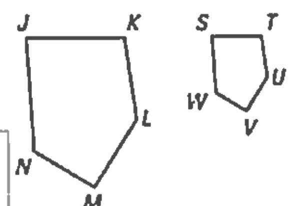
In the diagram, *ABCD* is similar to *EFGH*. The symbol ~ is used to indicate similarity. So, *ABCD* ~ *EFGH*.




$$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE}$$

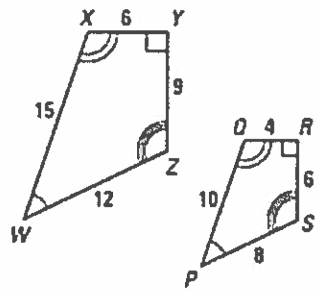
## EXAMPLE 1 *Writing Similarity Statements*

Pentagons *JKLMN* and *STUVW* are similar. List all the pairs of congruent angles. Write the ratios of the corresponding sides in a statement of proportionality.



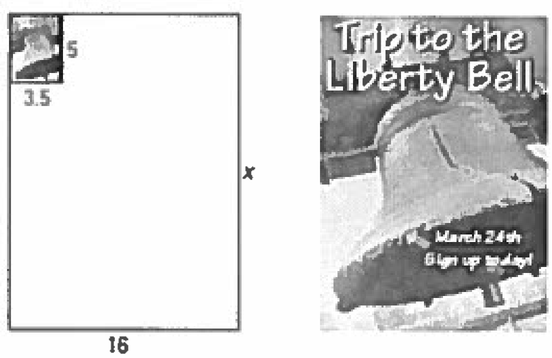
**EXAMPLE 2** ***Comparing Similar Polygons***

Decide whether the figures are similar.
If they are similar, write a similarity statement.



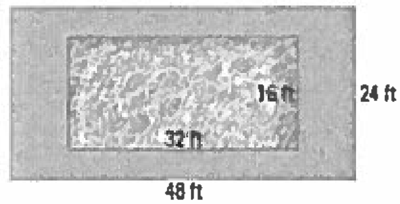

**EXAMPLE 3** ***Comparing Photographic Enlargements***

**POSTER DESIGN** You have been asked to create a poster to advertise a field trip to see the Liberty Bell. You have a 3.5 inch by 5 inch photo that you want to enlarge. You want the enlargement to be 16 inches wide. How long will it be?



**EXAMPLE 4** *Using Similar Polygons*

The rectangular patio around a pool is similar to the pool as shown. Calculate the scale factor of the patio to the pool, and find the ratio of their perimeters.




**SOLUTION**

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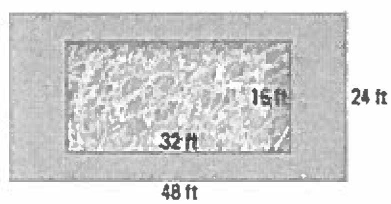



**SOLUTION**

Because the rectangles are similar, the scale factor of the patio to the pool is 48 ft : 32 ft, which is 3 : 2 in simplified form.

The perimeter of the patio is $2(24) + 2(48) = 144$ feet and the perimeter of the pool is $2(16) + 2(32) = 96$ feet. The ratio of the perimeters is $\frac{144}{96}$, or $\frac{3}{2}$.

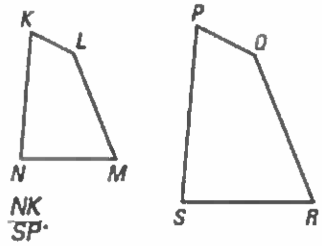
**THEOREM**

**THEOREM 8.1**

If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.

If $KLMN \sim PQRS$, then

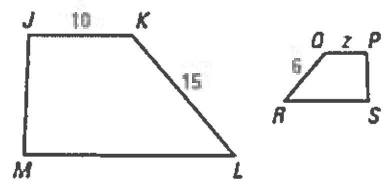
$$\frac{KL + LM + MN + NK}{PQ + QR + RS + SP} = \frac{KL}{PQ} = \frac{LM}{QR} = \frac{MN}{RS} = \frac{NK}{SP}.$$




**EXAMPLE 5** *Using Similar Polygons*

Quadrilateral *JKLM* is similar to quadrilateral *PQRS*.

Find the value of *z*.

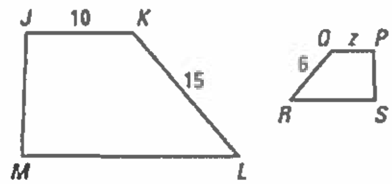



**SOLUTION**

## EXAMPLE 5 *Using Similar Polygons*

Quadrilateral $JKLM$ is similar to quadrilateral $PQRS$.

Find the value of $z$.




**SOLUTION**

Set up a proportion that contains $PQ$.

$\frac{KL}{QR} = \frac{JK}{PQ}$ **Write proportion.**

$\frac{15}{6} = \frac{10}{z}$ **Substitute.**

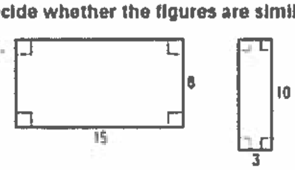
$z = 4$ **Cross multiply and divide by 15.**

## GUIDED PRACTICE

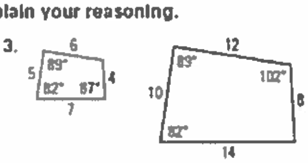
**Vocabulary Check** ✓ 1. If two polygons are similar, must they also be congruent? Explain.

**Concept Check** ✓ **Decide whether the figures are similar. Explain your reasoning.**

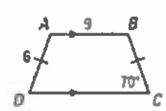
2.



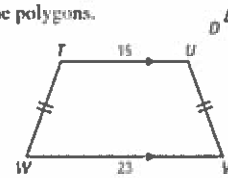

3.



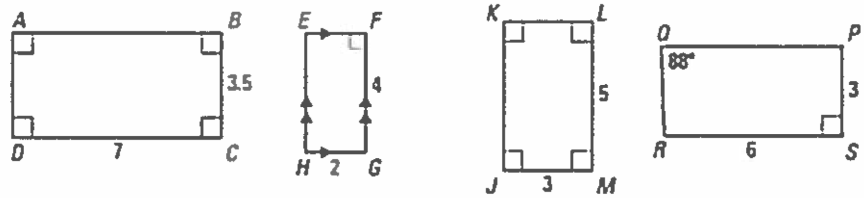

**Skill Check** ✓ **In the diagram, $TUVW \sim ABCD$.**




4. List all pairs of congruent angles and write the statement of proportionality for the polygons.
5. Find the scale factor of $TUVW$ to $ABCD$.
6. Find the length of $\overline{TW}$.
7. Find the measure of $\angle TUV$.



**DETERMINING SIMILARITY** Decide whether the quadrilaterals are similar. Explain your reasoning.




11. *ABCD* and *FGHE*

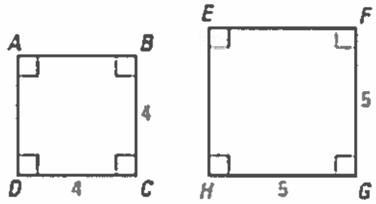
12. *ABCD* and *JKLM*

13. *ABCD* and *PQRS*

14. *JKLM* and *PQRS*

**DETERMINING SIMILARITY** Decide whether the polygons are similar. If so, write a similarity statement.

15.

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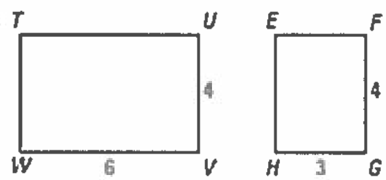


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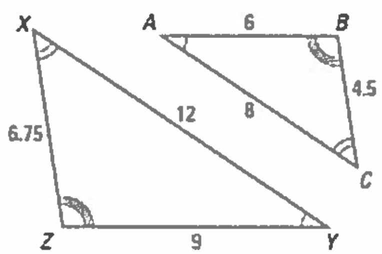


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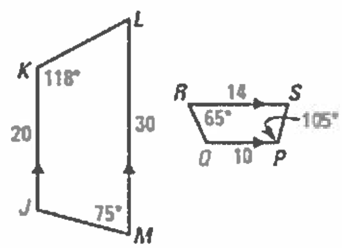
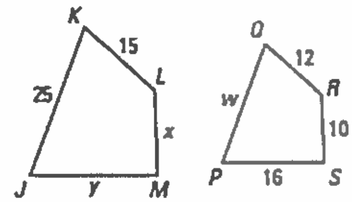


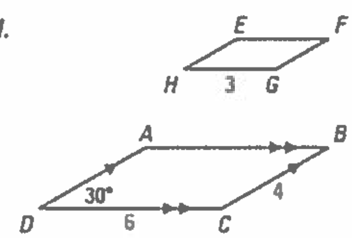
**USING SIMILAR POLYGONS** *PQRS* ~ *JKLM*.

19. Find the scale factor of *PQRS* to *JKLM*.
20. Find the scale factor of *JKLM* to *PQRS*.
21. Find the values of $w$, $x$, and $y$.
22. Find the perimeter of each polygon.
23. Find the ratio of the perimeter of *PQRS* to the perimeter of *JKLM*.



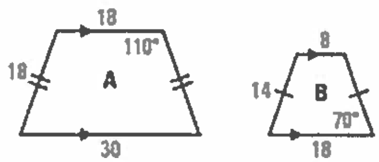

**USING SIMILAR POLYGONS** ▱*ABCD* ~ ▱*EFGH*.

24. Find the scale factor of ▱*ABCD* to ▱*EFGH*.
25. Find the length of $\overline{EH}$.
26. Find the measure of $\angle G$.
27. Find the perimeter of ▱*EFGH*.
28. Find the ratio of the perimeter of ▱*EFGH* to the perimeter of ▱*ABCD*.

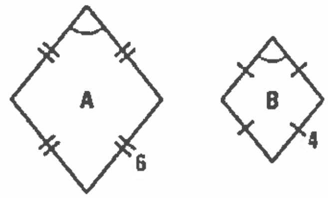


**DETERMINING SIMILARITY Decide whether the polygons are similar. If so, find the scale factor of Figure A to Figure B.**

**29.**

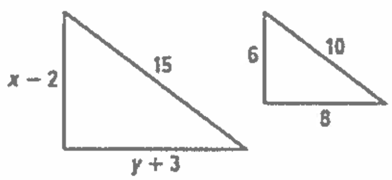



**30.**

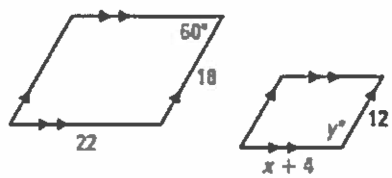



**USING ALGEBRA The two polygons are similar. Find the values of *x* and *y*.**

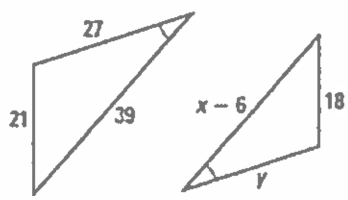
**39.**



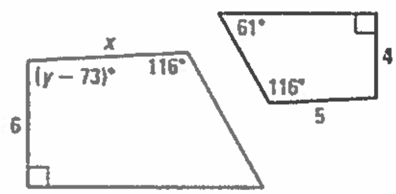

**40.**




**41.**




**42.**



# 8.4 Similar Triangles

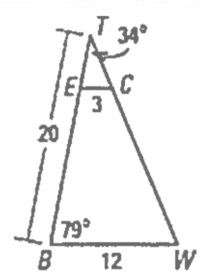
**EXAMPLE 1** *Writing Proportionality Statements*

In the diagram, $\triangle BTW \sim \triangle ETC$.

a. Write the statement of proportionality.

b. Find $m\angle TEC$.

c. Find $ET$ and $BE$.



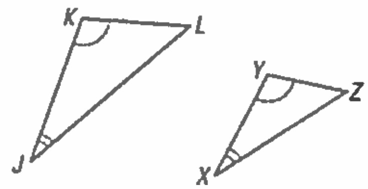

**SOLUTION**

## POSTULATE

**POSTULATE 25** ***Angle-Angle (AA) Similarity Postulate***

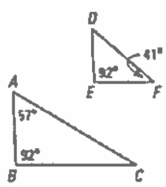
If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.

If $\angle JKL \cong \angle XYZ$ and $\angle KJL \cong \angle YXZ$, then $\triangle JKL \sim \triangle XYZ$.




**DETERMINING SIMILARITY** Determine whether the triangles can be proved similar. If they are similar, write a similarity statement. If they are not similar, explain why.

18.

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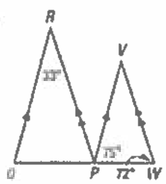


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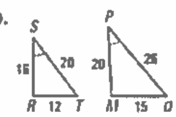


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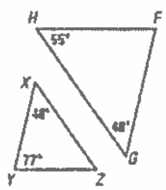


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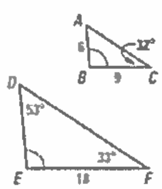


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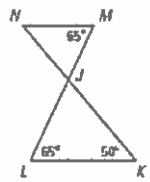


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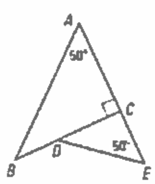


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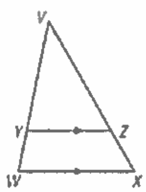


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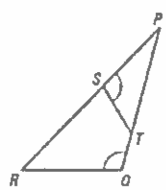
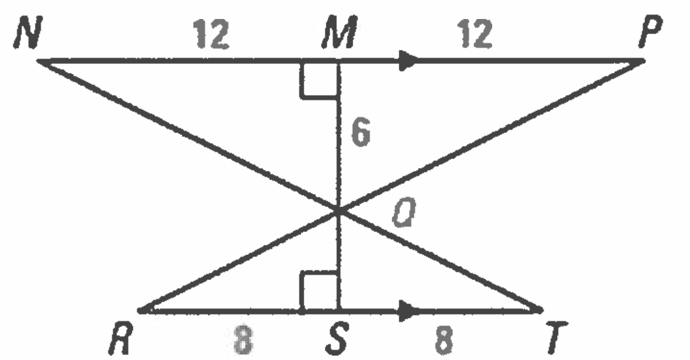


## EXAMPLE 5 *Using Scale Factors*

Find the length of the altitude $\overline{QS}$.




## GUIDED PRACTICE

**Vocabulary Check ✓**

1. If $\triangle ABC \sim \triangle XYZ$, $AB = 6$, and $XY = 4$, what is the *scale factor* of the triangles?

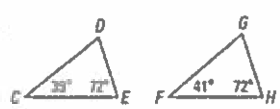
**Concept Check ✓**

2. The points $A(2, 3)$, $B(-1, 6)$, $C(4, 1)$, and $D(0, 5)$ lie on a line. Which two points could be used to calculate the slope of the line? Explain.
3. Can you assume that corresponding sides and corresponding angles of any two similar triangles are congruent?

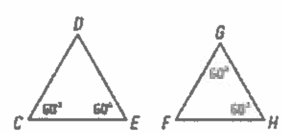
**Skill Check ✓**

**Determine whether $\triangle CDE \sim \triangle FGH$.**

4.

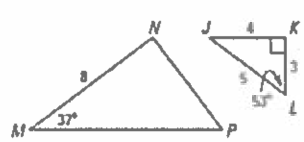



5.

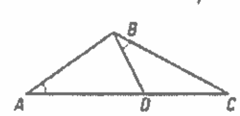



**In the diagram shown $\triangle JKL \sim \triangle MNP$.**

6. Find $m\angle J$, $m\angle N$, and $m\angle P$.
7. Find $MP$ and $PN$.

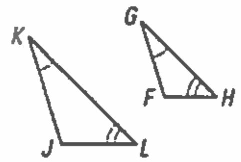



8. Given that $\angle CAB \cong \angle CBD$, how do you know that $\triangle ABC \sim \triangle BDC$? Explain your answer.



**USING SIMILARITY STATEMENTS** The triangles shown are similar. List all the pairs of congruent angles and write the statement of proportionality.

9.

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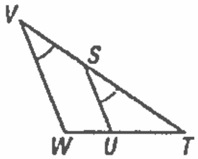
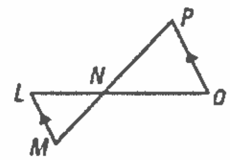



11.




**LOGICAL REASONING** Use the diagram to complete the following.

12. $\triangle PQR \sim$ __?__

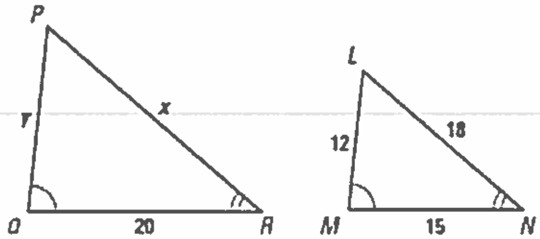
13. $\frac{PQ}{?} = \frac{QR}{?} = \frac{RP}{?}$

14. $\frac{20}{?} = \frac{?}{12}$

15. $\frac{?}{20} = \frac{18}{?}$

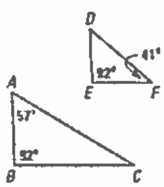
16. $y =$ __?__

17. $x =$ __?__




**DETERMINING SIMILARITY** Determine whether the triangles can be proved similar. If they are similar, write a similarity statement. If they are not similar, explain why.

18.

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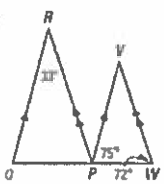


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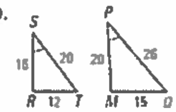


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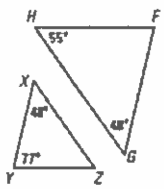


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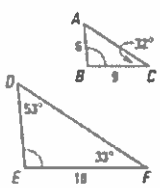


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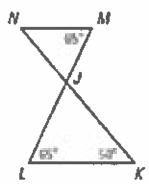


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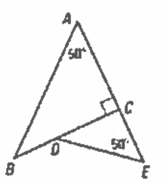


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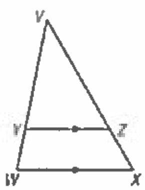


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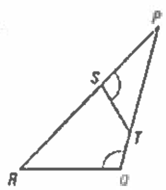


# 8.5 Proving Triangles are Similar

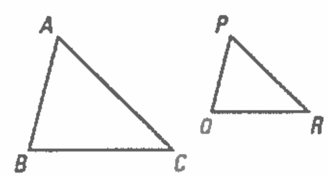
THEOREMS

**THEOREM 8.2** *Side-Side-Side (SSS) Similarity Theorem*

If the lengths of the corresponding sides of two triangles are proportional, then the triangles are similar.

If $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$,

then $\triangle ABC \sim \triangle PQR$.

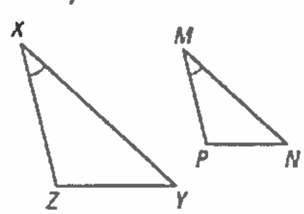



**THEOREM 8.3** *Side-Angle-Side (SAS) Similarity Theorem*

If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.

If $\angle X \cong \angle M$ and $\frac{ZX}{PM} = \frac{XY}{MN}$,

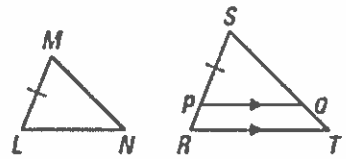
then $\triangle XYZ \sim \triangle MNP$.



**EXAMPLE 1** *Proof of Theorem 8.2*

**GIVEN** ▸ $\frac{RS}{LM} = \frac{ST}{MN} = \frac{TR}{NL}$

**PROVE** ▸ $\triangle RST \sim \triangle LMN$

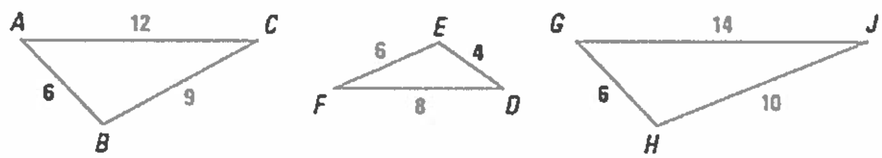



**SOLUTION**

***Paragraph Proof*** Locate $P$ on $\overline{RS}$ so that $PS = LM$. Draw $\overline{PQ}$ so that $\overline{PQ} \parallel \overline{RT}$. Then $\triangle RST \sim \triangle PSQ$, by the AA Similarity Postulate, and $\frac{RS}{PS} = \frac{ST}{SQ} = \frac{TR}{QP}$. Because $PS = LM$, you can substitute in the given proportion and find that $SQ = MN$ and $QP = NL$. By the SSS Congruence Theorem, it follows that $\triangle PSQ \cong \triangle LMN$. Finally, use the definition of congruent triangles and the AA Similarity Postulate to conclude that $\triangle RST \sim \triangle LMN$.

**EXAMPLE 2** *Using the SSS Similarity Theorem*

Which of the following three triangles are similar?



**EXAMPLE 2** *Using the SSS Similarity Theorem*

Which of the following three triangles are similar?


**SOLUTION**

To decide which, if any, of the triangles are similar, you need to consider the ratios of the lengths of corresponding sides.

*Ratios of Side Lengths of $\triangle ABC$ and $\triangle DEF$*

| $\frac{AB}{DE} = \frac{6}{4} = \frac{3}{2}$, | $\frac{CA}{FD} = \frac{12}{8} = \frac{3}{2}$, | $\frac{BC}{EF} = \frac{9}{6} = \frac{3}{2}$ |
|---|---|---|
| **Shortest sides** | **Longest sides** | **Remaining sides** |

▶ Because all of the ratios are equal, $\triangle ABC \sim \triangle DEF$.

*Ratios of Side Lengths of $\triangle ABC$ and $\triangle GHJ$*

| $\frac{AB}{GH} = \frac{6}{6} = 1$, | $\frac{CA}{JG} = \frac{12}{14} = \frac{6}{7}$, | $\frac{BC}{HJ} = \frac{9}{10}$ |
|---|---|---|
| **Shortest sides** | **Longest sides** | **Remaining sides** |

▶ Because the ratios are not equal, $\triangle ABC$ and $\triangle GHJ$ are not similar.

Since $\triangle ABC$ is similar to $\triangle DEF$ and $\triangle ABC$ is not similar to $\triangle GHJ$, $\triangle DEF$ is not similar to $\triangle GHJ$.

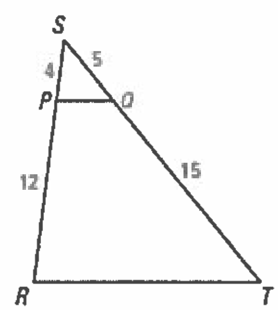
**EXAMPLE 3** *Using the SAS Similarity Theorem*

Use the given lengths to prove that $\triangle RST \sim \triangle PSQ$.

**SOLUTION**

**GIVEN** ▶ $SP = 4$, $PR = 12$, $SQ = 5$, $QT = 15$

**PROVE** ▶ $\triangle RST \sim \triangle PSQ$



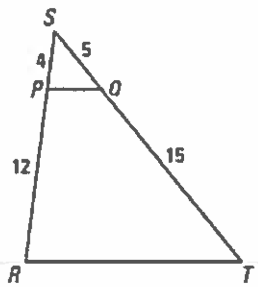
## EXAMPLE 3 *Using the SAS Similarity Theorem*

Use the given lengths to prove that $\triangle RST \sim \triangle PSQ$.

**SOLUTION**

**GIVEN** ▶ $SP = 4$, $PR = 12$, $SQ = 5$, $QT = 15$

**PROVE** ▶ $\triangle RST \sim \triangle PSQ$




***Paragraph Proof*** Use the SAS Similarity Theorem. Begin by finding the ratios of the lengths of the corresponding sides.

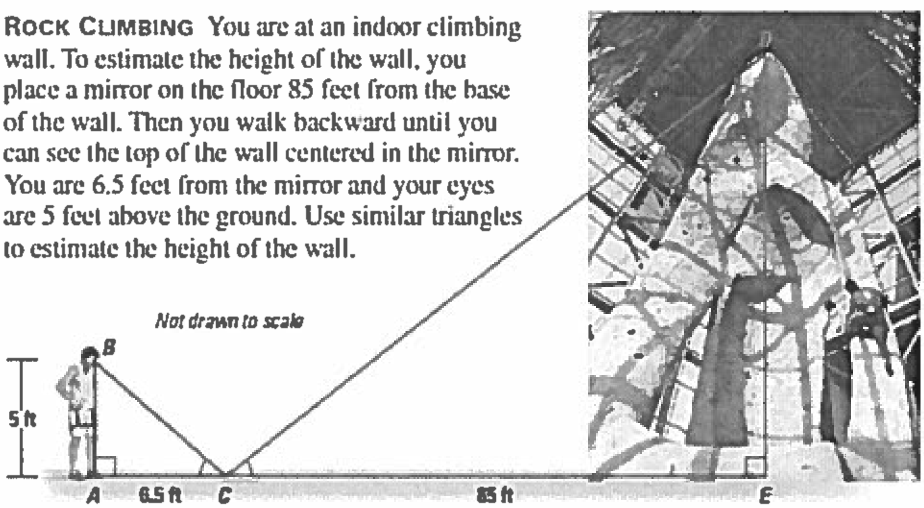
$$\frac{SR}{SP} = \frac{SP + PR}{SP} = \frac{4 + 12}{4} = \frac{16}{4} = 4$$

$$\frac{ST}{SQ} = \frac{SQ + QT}{SQ} = \frac{5 + 15}{5} = \frac{20}{5} = 4$$

So, the lengths of sides $\overline{SR}$ and $\overline{ST}$ are proportional to the lengths of the corresponding sides of $\triangle PSQ$. Because $\angle S$ is the included angle in both triangles, use the SAS Similarity Theorem to conclude that $\triangle RST \sim \triangle PSQ$.

## EXAMPLE 5 *Finding Distance Indirectly*

**ROCK CLIMBING** You are at an indoor climbing wall. To estimate the height of the wall, you place a mirror on the floor 85 feet from the base of the wall. Then you walk backward until you can see the top of the wall centered in the mirror. You are 6.5 feet from the mirror and your eyes are 5 feet above the ground. Use similar triangles to estimate the height of the wall.



**SOLUTION**

Due to the reflective property of mirrors, you can reason that $\angle ACB \cong \angle ECD$. Using the fact that $\triangle ABC$ and $\triangle EDC$ are right triangles, you can apply the AA Similarity Postulate to conclude that these two triangles are similar.

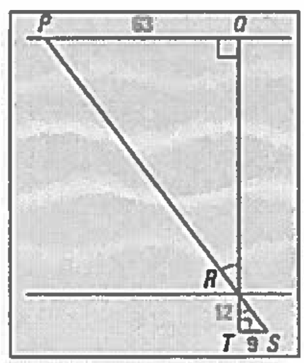
$\frac{DE}{BA} = \frac{EC}{AC}$ — **Ratios of lengths of corresponding sides are equal.**

$\frac{DE}{5} = \frac{85}{6.5}$ — **Substitute.**

$65.38 \approx DE$ — **Multiply each side by 5 and simplify.**

▶ So, the height of the wall is about 65 feet.

**INDIRECT MEASUREMENT** To measure the width of a river, you use a surveying technique, as shown in the diagram. Use the given lengths (measured in feet) to find $RQ$.




**SOLUTION**

By the AA Similarity Postulate, $\triangle PQR \sim \triangle STR$.

$\frac{RQ}{RT} = \frac{PQ}{ST}$ — **Write proportion.**

$\frac{RQ}{12} = \frac{63}{9}$ — **Substitute.**

$RQ = 12 \cdot 7$ — **Multiply each side by 12.**

$RQ = 84$ — **Simplify.**

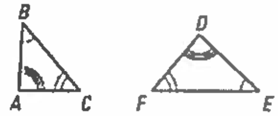
▶ So, the river is 84 feet wide.

1. You want to prove that $\triangle FHG$ is similar to $\triangle RXS$ by the SSS Similarity Theorem. Complete the proportion that is needed to use this theorem.

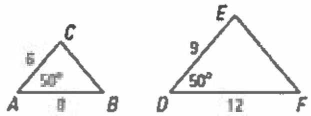
$$\frac{FH}{?} = \frac{?}{XS} = \frac{FG}{?}$$

**Name a postulate or theorem that can be used to prove that the two triangles are similar. Then, write a similarity statement.**

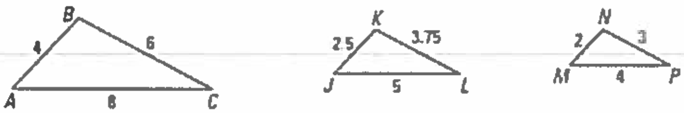
2.




3.



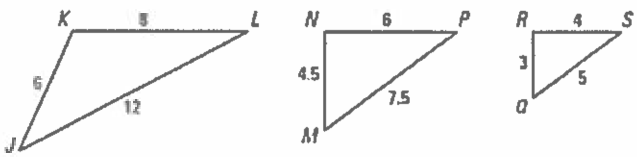

4. Which triangles are similar to $\triangle ABC$? Explain.




5. The side lengths of $\triangle ABC$ are 2, 5, and 6, and $\triangle DEF$ has side lengths of 12, 30, and 36. Find the ratios of the lengths of the corresponding sides of $\triangle ABC$ to $\triangle DEF$. Are the two triangles similar? Explain.

**DETERMINING SIMILARITY In Exercises 6–8, determine which two of the three given triangles are similar. Find the scale factor for the pair.**

6.

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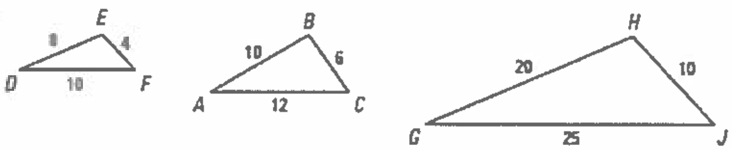


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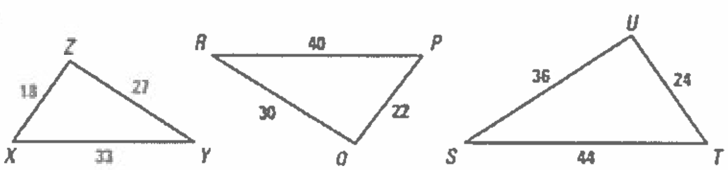
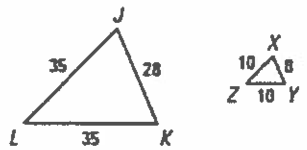


**DETERMINING SIMILARITY** **Are the triangles similar? If so, state the similarity and the postulate or theorem that justifies your answer.**

9.

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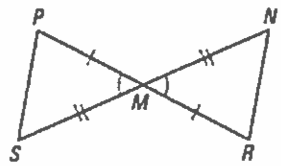


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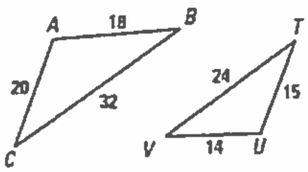


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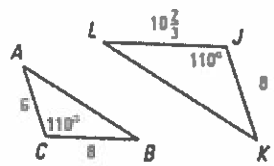


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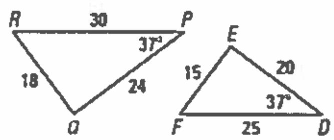


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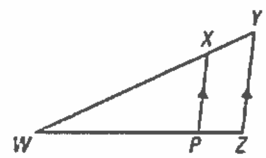



**LOGICAL REASONING** **Draw the given triangles roughly to scale. Then, name a postulate or theorem that can be used to prove that the triangles are similar.**

15. The side lengths of $\triangle PQR$ are 16, 8, and 18, and the side lengths of $\triangle XYZ$ are 9, 8, and 4.

16. In $\triangle ABC$, $m\angle A = 28°$ and $m\angle B = 62°$. In $\triangle DEF$, $m\angle D = 28°$ and $m\angle F = 90°$.

17. In $\triangle STU$, the length of $\overline{ST}$ is 18, the length of $\overline{SU}$ is 24, and $m\angle S = 65°$. The length of $\overline{JK}$ is 6, $m\angle J = 65°$, and the length of $\overline{JL}$ is 8 in $\triangle JKL$.

18. The ratio of $VW$ to $MN$ is 6 to 1. In $\triangle VWX$, $m\angle W = 30°$, and in $\triangle MNP$, $m\angle N = 30°$. The ratio of $WX$ to $NP$ is 6 to 1.

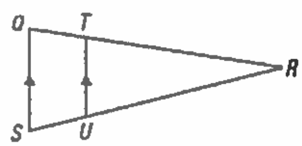
# 8.6 Proportions and Similar Triangles

**THEOREMS**

**THEOREM 8.4** *Triangle Proportionality Theorem*

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

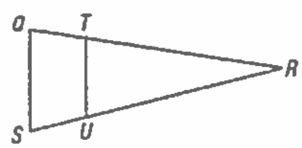
If $\overline{TU} \parallel \overline{QS}$, then $\frac{RT}{TQ} = \frac{RU}{US}$.




**THEOREM 8.5** *Converse of the Triangle Proportionality Theorem*

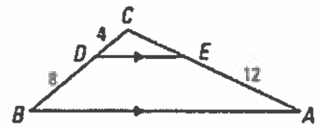
If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

If $\frac{RT}{TQ} = \frac{RU}{US}$, then $\overline{TU} \parallel \overline{QS}$.



**EXAMPLE 1** *Finding the Length of a Segment*

In the diagram $\overline{AB} \parallel \overline{ED}$.
$BD = 8$, $DC = 4$, and $AE = 12$.
What is the length of $\overline{EC}$?




**EXAMPLE 1** *Finding the Length of a Segment*

In the diagram $\overline{AB} \parallel \overline{ED}$.
$BD = 8$, $DC = 4$, and $AE = 12$.
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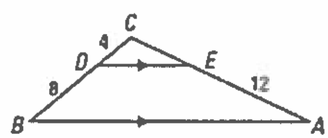



**SOLUTION**

$\frac{DC}{BD} = \frac{EC}{AE}$ Triangle Proportionality Theorem

$\frac{4}{8} = \frac{EC}{12}$ Substitute.

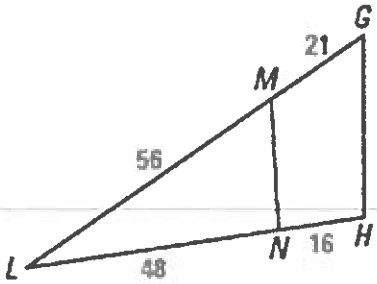
$\frac{4(12)}{8} = EC$ Multiply each side by 12.

$6 = EC$ Simplify.

▶ So, the length of $\overline{EC}$ is 6.

EXAMPLE 2 **Determining Parallels**

Given the diagram, determine whether $MN \parallel GH$.




EXAMPLE 2 **Determining Parallels**

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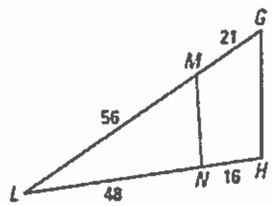



**SOLUTION**

Begin by finding and simplifying the ratios of the two sides divided by $MN$.

$$\frac{LM}{MG} = \frac{56}{21} = \frac{8}{3} \qquad \frac{LN}{NH} = \frac{48}{16} = \frac{3}{1}$$

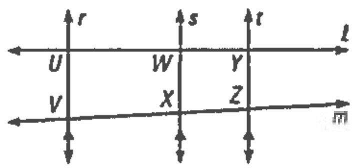
Because $\frac{8}{3} \neq \frac{3}{1}$, $MN$ is not parallel to $GH$.

**THEOREMS**

**THEOREM 8.6**

If three parallel lines intersect two transversals, then they divide the transversals proportionally.

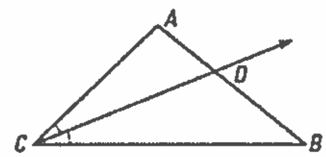
If $r \parallel s$ and $s \parallel t$, and $\ell$ and $m$ intersect $r$, $s$, and $t$, then $\frac{UW}{WY} = \frac{VX}{XZ}$.




**THEOREM 8.7**

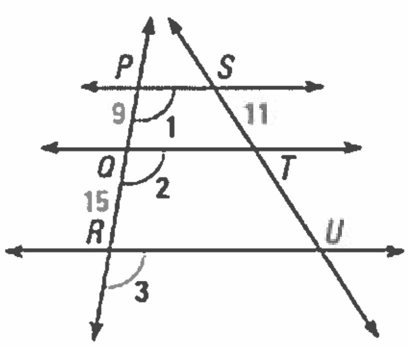
If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.

If $\overrightarrow{CD}$ bisects $\angle ACB$, then $\frac{AD}{DB} = \frac{CA}{CB}$.



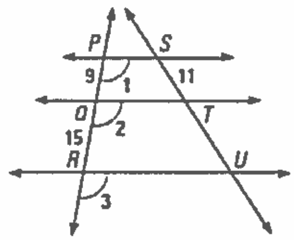

## EXAMPLE 3 *Using Proportionality Theorems*

In the diagram, $\angle 1 \cong \angle 2 \cong \angle 3$, and $PQ = 9$, $QR = 15$, and $ST = 11$. What is the length of $\overline{TU}$?



**EXAMPLE 3** *Using Proportionality Theorems*

In the diagram, $\angle 1 \cong \angle 2 \cong \angle 3$, and $PQ = 9$, $QR = 15$, and $ST = 11$. What is the length of $\overline{TU}$?




**SOLUTION**

Because corresponding angles are congruent the lines are parallel and you can use Theorem 8.6.

$\frac{PQ}{QR} = \frac{ST}{TU}$ **Parallel lines divide transversals proportionally.**

$\frac{9}{15} = \frac{11}{TU}$ **Substitute.**

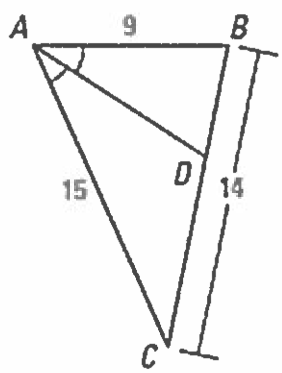
$9 \cdot TU = 15 \cdot 11$ **Cross product property**

$TU = \frac{15(11)}{9} = \frac{55}{3}$ **Divide each side by 9 and simplify.**

▶ So, the length of $\overline{TU}$ is $\frac{55}{3}$, or $18\frac{1}{3}$.

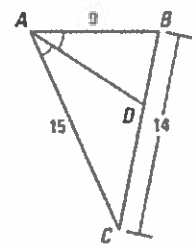
**EXAMPLE 4** *Using Proportionality Theorems*

In the diagram, $\angle CAD \cong \angle DAB$. Use the given side lengths to find the length of $\overline{DC}$.



**EXAMPLE 4** *Using Proportionality Theorems*

In the diagram, $\angle CAD \cong \angle DAB$. Use the given side lengths to find the length of $\overline{DC}$.




**SOLUTION**

Since $\overline{AD}$ is an angle bisector of $\angle CAB$, you can apply Theorem 8.7.

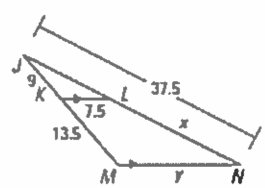
Let $x = DC$. Then, $BD = 14 - x$.

| | |
|---|---|
| $\frac{AB}{AC} = \frac{BD}{DC}$ | Apply Theorem 8.7. |
| $\frac{9}{15} = \frac{14 - x}{x}$ | Substitute. |
| $9 \cdot x = 15(14 - x)$ | Cross product property |
| $9x = 210 - 15x$ | Distributive property |
| $24x = 210$ | Add $15x$ to each side. |
| $x = 8.75$ | Divide each side by 24. |

▶ So, the length of $\overline{DC}$ is 8.75 units.

**EXAMPLE 6** *Finding Segment Lengths*

In the diagram $\overline{KL} \parallel \overline{MN}$. Find the values of the variables.




**SOLUTION**

To find the value of $x$, you can set up a proportion.

| | |
|---|---|
| $\frac{9}{13.5} = \frac{37.5 - x}{x}$ | Write proportion. |
| $13.5(37.5 - x) = 9x$ | Cross product property |
| $506.25 - 13.5x = 9x$ | Distributive property |
| $506.25 = 22.5x$ | Add $13.5x$ to each side. |
| $22.5 = x$ | Divide each side by 22.5. |

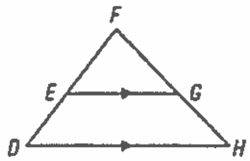
Since $\overline{KL} \parallel \overline{MN}$, $\triangle JKL \sim \triangle JMN$ and $\frac{JK}{JM} = \frac{KL}{MN}$.

| | |
|---|---|
| $\frac{9}{13.5 + 9} = \frac{7.5}{y}$ | Write proportion. |
| $9y = 7.5(22.5)$ | Cross product property |
| $y = 18.75$ | Divide each side by 9. |

1. Complete the following: If a line divides two sides of a triangle proportionally, then it is __?__ to the third side. This theorem is known as the __?__.

2. In $\triangle ABC$, $\overrightarrow{AR}$ bisects $\angle CAB$. Write the proportionality statement for the triangle that is based on Theorem 8.7.

**Determine whether the statement is *true* or *false*. Explain your reasoning.**



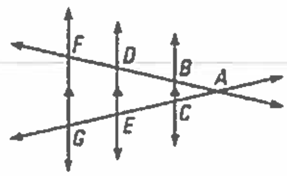

3. $\frac{FE}{ED} = \frac{FG}{GH}$

4. $\frac{FE}{FD} = \frac{FG}{FH}$

5. $\frac{EG}{DH} = \frac{EF}{DF}$

6. $\frac{ED}{FE} = \frac{EG}{DH}$

**Use the figure to complete the proportion.**




7. $\frac{BD}{BF} = \frac{?}{CG}$

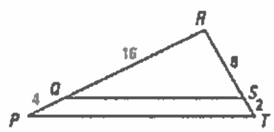
8. $\frac{AE}{CE} = \frac{?}{BD}$

9. $\frac{?}{GA} = \frac{FD}{FA}$

10. $\frac{GA}{?} = \frac{FA}{DA}$

**LOGICAL REASONING** Determine whether the given information implies that $\overline{QS} \parallel \overline{PT}$. Explain.

11.

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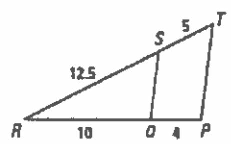


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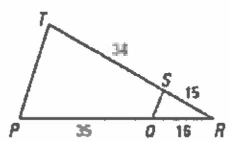


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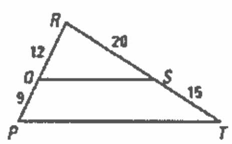
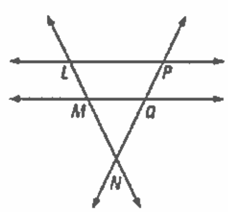



**LOGICAL REASONING** Use the diagram shown to decide if you are given enough information to conclude that $\overline{LP} \parallel \overline{MQ}$. If so, state the reason.




15. $\frac{NM}{ML} = \frac{NQ}{QP}$

16. $\angle MNQ \cong \angle LNP$

17. $\angle NLP \cong \angle NMQ$

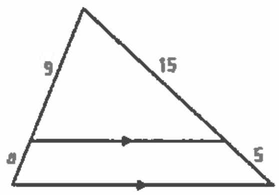
18. $\angle MQN \cong \angle LPN$

19. $\frac{LM}{MN} = \frac{LP}{MQ}$

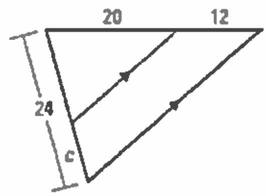
20. $\triangle LPN \sim \triangle MQN$

USING PROPORTIONALITY THEOREMS **Find the value of the variable.**

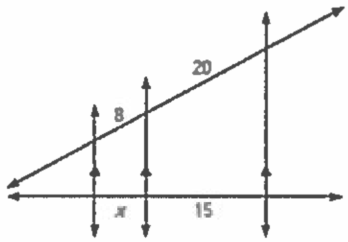
**21.**



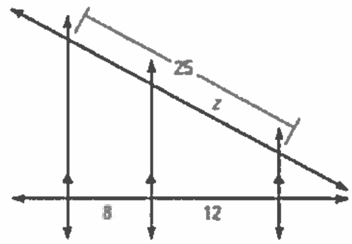

**22.**




**23.**

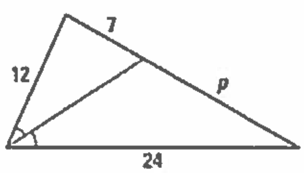



**24.**

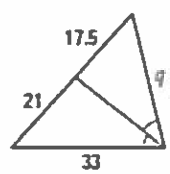



USING ALGEBRA **Find the value of the variable.**

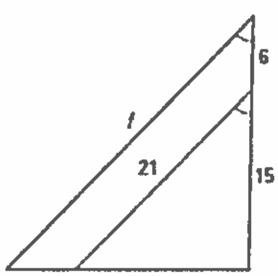
**25.**



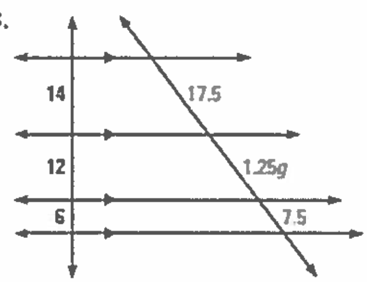

**26.**




**27.**




**28.**

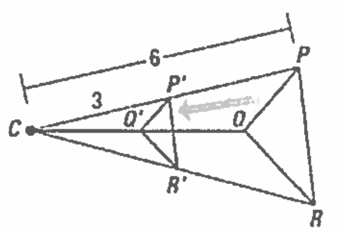


# 8.7 Dilations

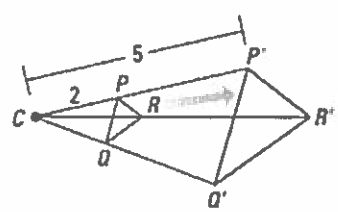
A **dilation** with center $C$ and scale factor $k$ is a transformation that maps every point $P$ in the plane to a point $P'$ so that the following properties are true.

1. If $P$ is not the center point $C$, then the image point $P'$ lies on $\overrightarrow{CP}$. The scale factor $k$ is a positive number such that $k = \frac{CP'}{CP}$, and $k \neq 1$.
2. If $P$ is the center point $C$, then $P = P'$.

The dilation is a **reduction** if $0 < k < 1$ and it is an **enlargement** if $k > 1$.




Reduction: $k = \frac{CP'}{CP} = \frac{3}{6} = \frac{1}{2}$

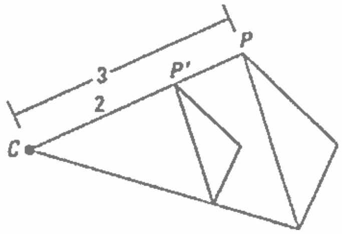



Enlargement: $k = \frac{CP'}{CP} = \frac{5}{2}$

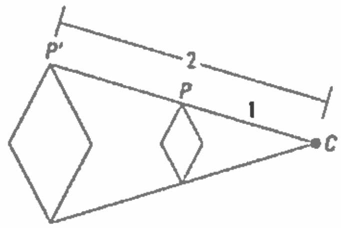
Because $\triangle PQR \sim \triangle P'Q'R'$, $\frac{P'Q'}{PQ}$ is equal to the scale factor of the dilation.

Identify the dilation and find its scale factor.

a.

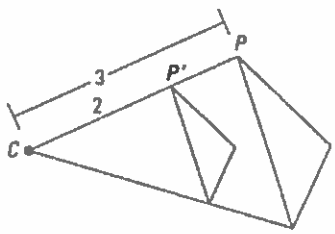



b.

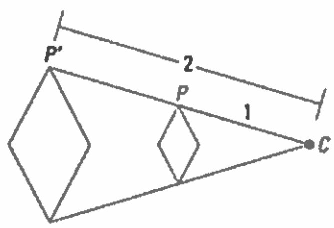



Identify the dilation and find its scale factor.

a.




b.



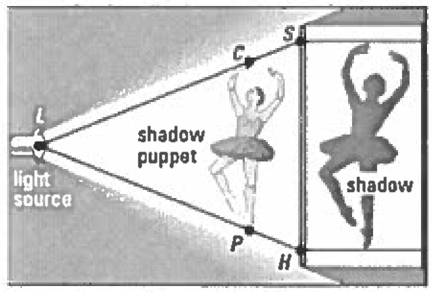

**SOLUTION**

a. Because $\frac{CP'}{CP} = \frac{2}{3}$, the scale factor is $k = \frac{2}{3}$. This is a reduction.

b. Because $\frac{CP'}{CP} = \frac{2}{1}$, the scale factor is $k = 2$. This is an enlargement.

**EXAMPLE 3** *Finding the Scale Factor*

**SHADOW PUPPETS** Shadow puppets have been used in many countries for hundreds of years. A flat figure is held between a light and a screen. The audience on the other side of the screen sees the puppet's shadow. The shadow is a dilation, or enlargement, of the shadow puppet. When looking at a cross sectional view, $\triangle LCP \sim \triangle LSH$.




The shadow puppet shown is 12 inches tall ($CP$ in the diagram). Find the height of the shadow, $SH$, for each distance from the screen. In each case, by what percent is the shadow larger than the puppet?

**a.** $LC = LP = 59$ in.; $LS = LH = 74$ in.

**b.** $LC = LP = 66$ in.; $LS = LH = 74$ in.

**SOLUTION**

**a.** $\frac{59}{74} = \frac{12}{SH}$ $\qquad \frac{LC}{LS} = \frac{CP}{SH}$

$59(SH) = 888$

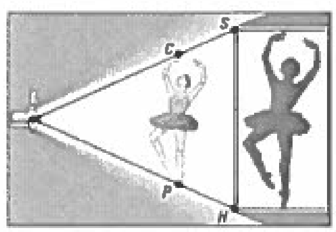
$SH \approx 15$ inches

To find the percent of size increase, use the scale factor of the dilation.

$\text{scale factor} = \frac{SH}{CP}$

$\frac{15}{12} = 1.25$

▶ So, the shadow is 25% larger than the puppet.




**b.** $\frac{66}{74} = \frac{12}{SH}$

$66(SH) = 888$

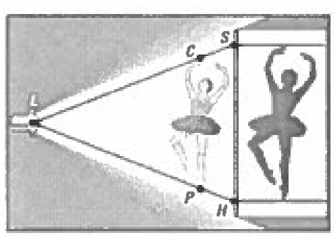
$SH \approx 13.45$ inches

Use the scale factor again to find the percent of size increase.

$\text{scale factor} = \frac{SH}{CP}$

$\frac{13.45}{12} \approx 1.12$

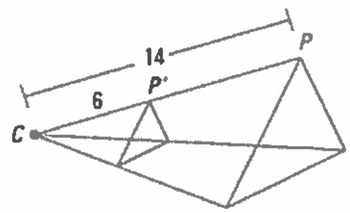
▶ So, the shadow is about 12% larger than the puppet.



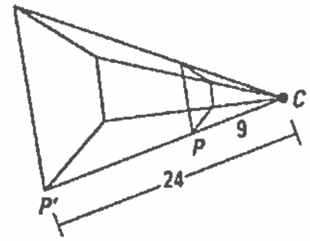

Notice that as the puppet moves closer to the screen, the shadow height decreases.

**IDENTIFYING DILATIONS** **Identify the dilation and find its scale factor.**

**8.**

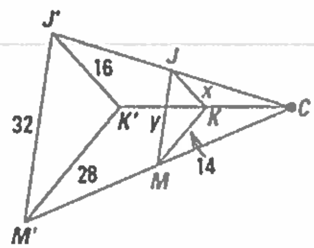



**9.**

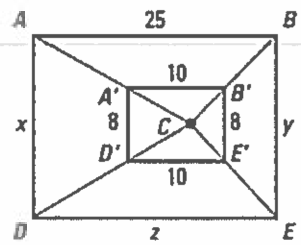



**FINDING SCALE FACTORS** **Identify the dilation, and find its scale factor. Then, find the values of the variables.**

**10.**

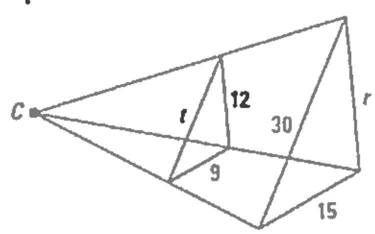



**11.**




**SIMILAR TRIANGLES** **The red triangle is the image of the blue triangle after a dilation. Find the values of the variables. Then find the ratio of their perimeters.**

**20.**




**21.**