

3.6

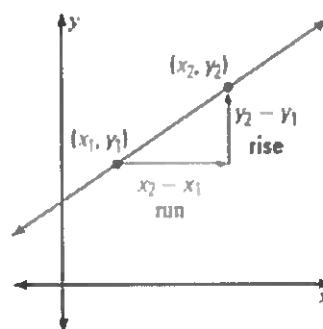
Parallel Lines in the Coordinate Plane

In algebra, you learned that the slope of a nonvertical line is the ratio of the vertical change (the rise) to the horizontal change (the run). If the line passes through the points (x_1, y_1) and (x_2, y_2) , then the slope is given by

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

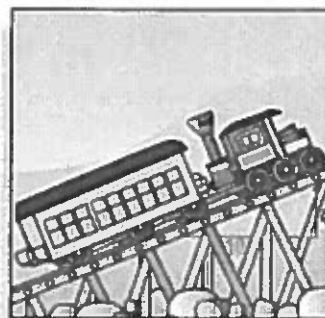
$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

Slope is usually represented by the variable m .



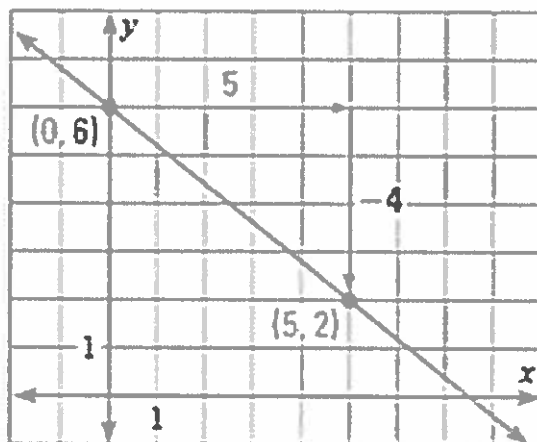
EXAMPLE 1 Finding the Slope of Train Tracks

COG RAILWAY A cog railway goes up the side of Mount Washington, the tallest mountain in New England. At the steepest section, the train goes up about 4 feet for each 10 feet it goes forward. What is the slope of this section?

**SOLUTION**

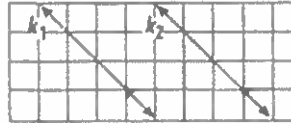
$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{4 \text{ feet}}{10 \text{ feet}} = 0.4$$

Find the slope of the line that passes through the points $(0, 6)$ and $(5, 2)$.



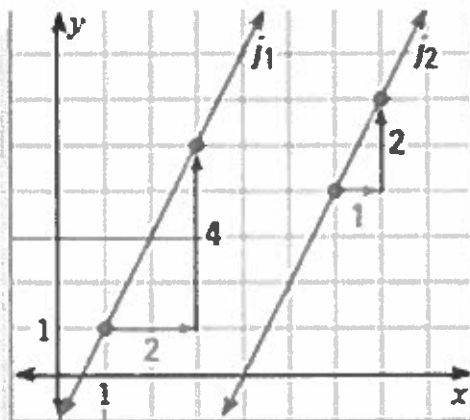
POSTULATE**POSTULATE 17** *Slopes of Parallel Lines*

In a coordinate plane, two nonvertical lines are parallel if and only if they have the same slope. Any two vertical lines are parallel.

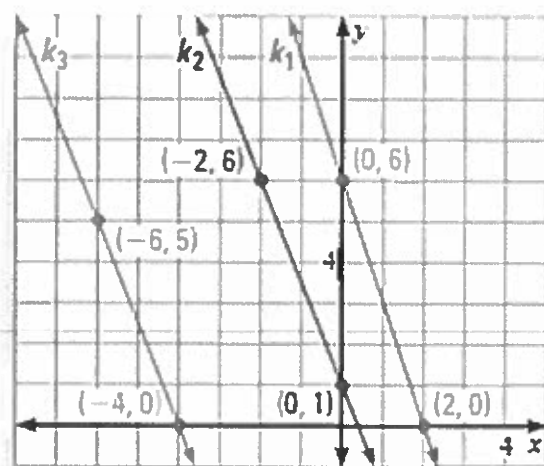


Lines k_1 and k_2 have the same slope.

Find the slope of each line. Is $j_1 \parallel j_2$?



Find the slope of each line. Which lines are parallel?



GOAL 2 WRITING EQUATIONS OF PARALLEL LINES

In algebra, you learned that you can use the slope m of a nonvertical line to write an equation of the line in *slope-intercept form*.

$$y = \overset{\text{slope}}{m}x + \overset{\text{y-intercept}}{b}$$

The y-intercept is the y-coordinate of the point where the line crosses the y-axis.

EXAMPLE 5 Writing an Equation of a Line

Write an equation of the line through the point $(2, 3)$ that has a slope of 5.

SOLUTION

Solve for b . Use $(x, y) = (2, 3)$ and $m = 5$.

$$y = mx + b \quad \text{Slope-intercept form}$$

$$3 = 5(2) + b \quad \text{Substitute 2 for } x, 3 \text{ for } y, \text{ and 5 for } m.$$

$$3 = 10 + b \quad \text{Simplify.}$$

$$-7 = b \quad \text{Subtract.}$$

► Write an equation. Since $m = 5$ and $b = -7$, an equation of the line is $y = 5x - 7$.

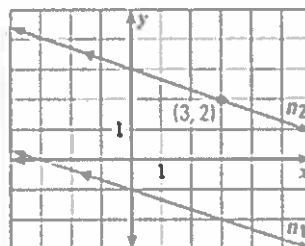
EXAMPLE 6 Writing an Equation of a Parallel Line

Line n_1 has the equation $y = -\frac{1}{3}x - 1$.

Line n_2 is parallel to n_1 and passes through the point $(3, 2)$. Write an equation of n_2 .

SOLUTION

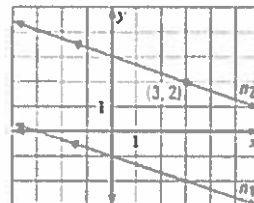
Find the slope.



EXAMPLE 6 Writing an Equation of a Parallel Line

Line n_1 has the equation $y = -\frac{1}{3}x - 1$.

Line n_2 is parallel to n_1 and passes through the point $(3, 2)$. Write an equation of n_2 .

**SOLUTION**

Find the slope.

The slope of n_1 is $-\frac{1}{3}$. Because parallel lines have the same slope, the slope of n_2 is also $-\frac{1}{3}$.

Solve for b . Use $(x, y) = (3, 2)$ and $m = -\frac{1}{3}$.

$$y = mx + b$$

$$2 = -\frac{1}{3}(3) + b$$

$$2 = -1 + b$$

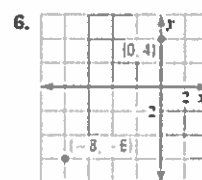
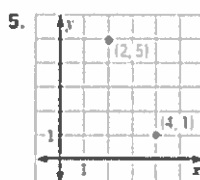
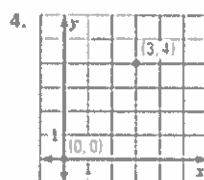
$$3 = b$$

Write an equation.

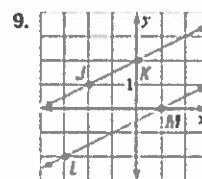
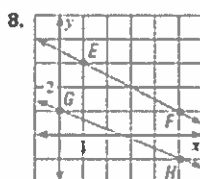
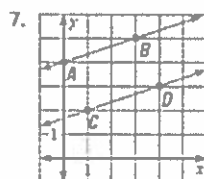
► Because $m = -\frac{1}{3}$ and $b = 3$, an equation of n_2 is $y = -\frac{1}{3}x + 3$.

1. What does *intercept* mean in the expression *slope-intercept form*?
2. The slope of line j is 2 and $j \parallel k$. What is the slope of line k ?
3. What is the slope of a horizontal line? What is the slope of a vertical line?

Find the slope of the line that passes through the labeled points.



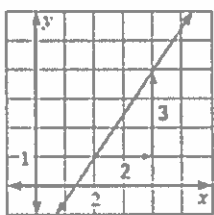
Determine whether the two lines shown in the graph are parallel. If they are parallel, explain how you know.



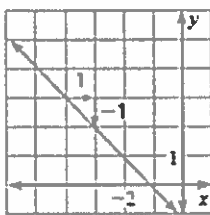
10. Write an equation of the line that passes through the point $(2, -3)$ and has a slope of -1 .

CALCULATING SLOPE What is the slope of the line?

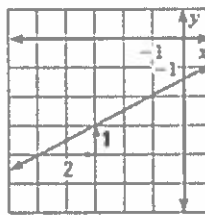
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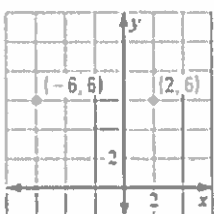
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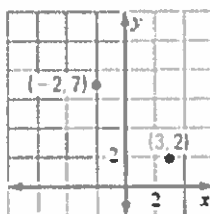
13.

**CALCULATING SLOPE** Find the slope of the line that passes through the labeled points on the graph.

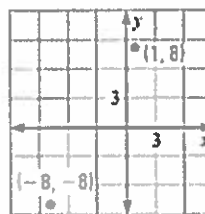
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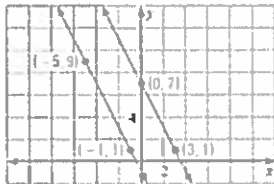
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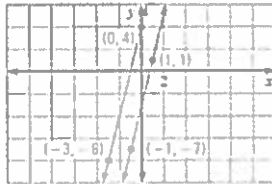
16.

**IDENTIFYING PARALLELS** Find the slope of each line. Are the lines parallel?

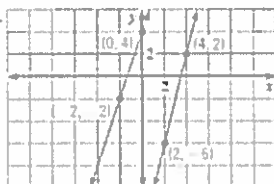
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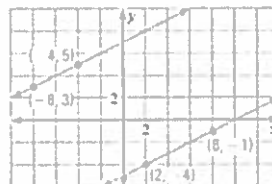
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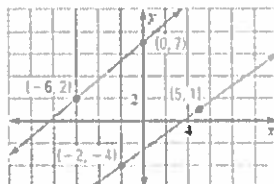
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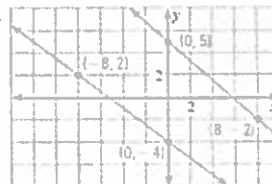
20.



21.



22.



WRITING EQUATIONS Write an equation of the line that passes through the given point P and has the given slope.

36. $P(0, -6), m = -2$

37. $P(-3, 9), m = -1$

38. $P\left(\frac{3}{2}, 4\right), m = \frac{1}{2}$

39. $P(2, -4), m = 0$

40. $P(-7, -5), m = \frac{3}{4}$

41. $P(6, 1)$, undefined slope

3.7

Perpendicular Lines in the Coordinate Plane

POSTULATE

POSTULATE 18 *Slopes of Perpendicular Lines*

In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is -1 .

Vertical and horizontal lines are perpendicular.



$$\text{product of slopes} = 2\left(-\frac{1}{2}\right) = -1$$

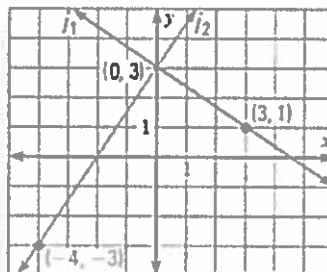
EXAMPLE 1 *Deciding Whether Lines are Perpendicular**Find each slope.*

$$\text{Slope of } j_1 = \frac{3 - 1}{0 - 3} = -\frac{2}{3}$$

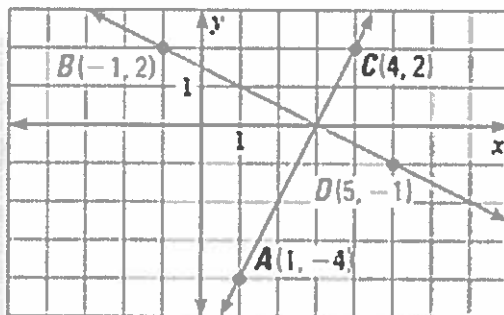
$$\text{Slope of } j_2 = \frac{3 - (-3)}{0 - (-4)} = \frac{6}{4} = \frac{3}{2}$$

Multiply the slopes.

The product is $\left(-\frac{2}{3}\right)\left(\frac{3}{2}\right) = -1$, so $j_1 \perp j_2$.



Decide whether \overleftrightarrow{AC} and \overleftrightarrow{DB} are perpendicular.



EXAMPLE 3 *Deciding Whether Lines are Perpendicular*

Decide whether the lines are perpendicular.

line h : $y = \frac{3}{4}x + 2$

line j : $y = -\frac{4}{3}x - 3$

EXAMPLE 4 *Deciding Whether Lines are Perpendicular*

Decide whether the lines are perpendicular.

line r : $4x + 5y = 2$

line s : $5x + 4y = 3$

GOAL 2 WRITING EQUATIONS OF PERPENDICULAR LINES**EXAMPLE 5** Writing the Equation of a Perpendicular Line

Line ℓ_1 has equation $y = -2x + 1$. Find an equation of the line ℓ_2 that passes through $P(4, 0)$ and is perpendicular to ℓ_1 . First you must find the slope, m_2 .

$$m_1 \cdot m_2 = -1 \quad \text{The product of the slopes of } \perp \text{ lines is } -1.$$

$$-2 \cdot m_2 = -1 \quad \text{The slope of } \ell_1 \text{ is } -2.$$

$$m_2 = \frac{1}{2} \quad \text{Divide both sides by } -2.$$

Then use $m = \frac{1}{2}$ and $(x, y) = (4, 0)$ to find b .

$$y = mx + b \quad \text{Slope-intercept form}$$

$$0 = \frac{1}{2}(4) + b \quad \text{Substitute 0 for } y, \frac{1}{2} \text{ for } m, \text{ and 4 for } x.$$

$$-2 = b \quad \text{Simplify.}$$

► So, an equation of ℓ_2 is $y = \frac{1}{2}x - 2$.

.....

**EXAMPLE 6** Writing the Equation of a Perpendicular Line

The equation $y = \frac{3}{2}x + 3$ represents a mirror. A ray of light hits the mirror at $(-2, 0)$. What is the equation of the line p that is perpendicular to the mirror at this point?

SOLUTION

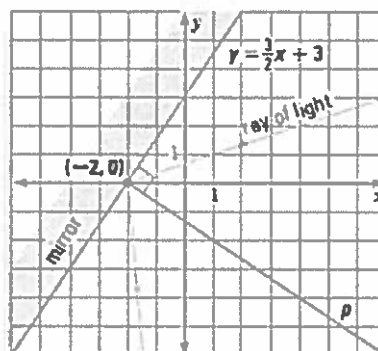
The mirror's slope is $\frac{3}{2}$, so the slope of p is $-\frac{2}{3}$.

Use $m = -\frac{2}{3}$ and $(x, y) = (-2, 0)$ to find b .

$$0 = -\frac{2}{3}(-2) + b$$

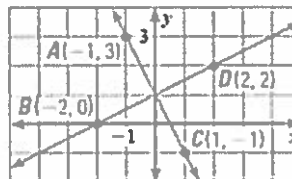
$$-\frac{4}{3} = b$$

► So, an equation for p is $y = -\frac{2}{3}x - \frac{4}{3}$.



Top view of mirror

1. Define *slope of a line*.
2. The slope of line m is $-\frac{1}{5}$. What is the slope of a line perpendicular to m ?
3. In the coordinate plane shown at the right, is \overrightarrow{AC} perpendicular to \overrightarrow{BD} ? Explain.
4. Decide whether the lines with the equations $y = 2x - 1$ and $y = -2x + 1$ are perpendicular.
5. Decide whether the lines with the equations $5y - x = 15$ and $y + 5x = 2$ are perpendicular.
6. The line ℓ_1 has the equation $y = 3x$. The line ℓ_2 is perpendicular to ℓ_1 and passes through the point $P(0, 0)$. Write an equation of ℓ_2 .



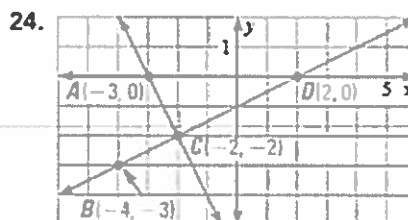
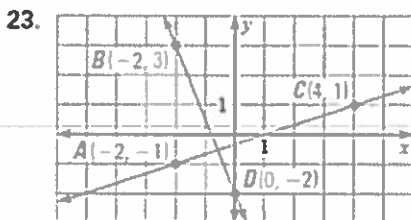
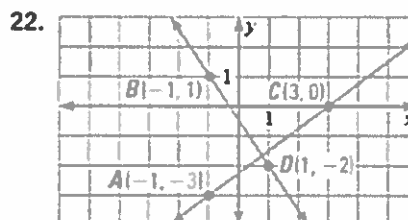
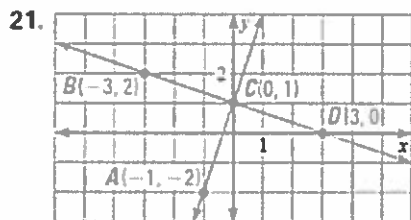
SLOPES OF PERPENDICULAR LINES The slopes of two lines are given. Are the lines perpendicular?

7. $m_1 = 2, m_2 = -\frac{1}{2}$
8. $m_1 = \frac{2}{3}, m_2 = \frac{3}{2}$
9. $m_1 = \frac{1}{4}, m_2 = -4$
10. $m_1 = \frac{5}{7}, m_2 = -\frac{7}{5}$
11. $m_1 = -\frac{1}{2}, m_2 = -\frac{1}{2}$
12. $m_1 = -1, m_2 = 1$

SLOPES OF PERPENDICULAR LINES Lines j and n are perpendicular. The slope of line j is given. What is the slope of line n ? Check your answer.

13. 2
14. 5
15. -3
16. -7
17. $\frac{2}{3}$
18. $\frac{1}{5}$
19. $-\frac{1}{3}$
20. $-\frac{4}{3}$

IDENTIFYING PERPENDICULAR LINES Find the slope of \overrightarrow{AC} and \overrightarrow{BD} .
Decide whether \overrightarrow{AC} is perpendicular to \overrightarrow{BD} .



34 USING ALGEBRA Decide whether lines k_1 and k_2 are perpendicular. Then graph the lines to check your answer.

25. line k_1 : $y = 3x$

line k_2 : $y = -\frac{1}{3}x - 2$

26. line k_1 : $y = -\frac{4}{5}x - 2$

line k_2 : $y = \frac{1}{5}x + 4$

27. line k_1 : $y = -\frac{3}{4}x + 2$

line k_2 : $y = \frac{4}{3}x + 5$

28. line k_1 : $y = \frac{1}{3}x - 10$

line k_2 : $y = 3x$

35 USING ALGEBRA Decide whether lines p_1 and p_2 are perpendicular.

29. line p_1 : $3y - 4x = 3$

line p_2 : $4y + 3x = -12$

30. line p_1 : $y - 6x = 2$

line p_2 : $6y - x = 12$

31. line p_1 : $3y + 2x = -36$

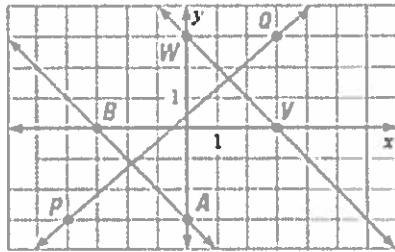
line p_2 : $4y - 3x = 16$

32. line p_1 : $5y + 3x = -15$

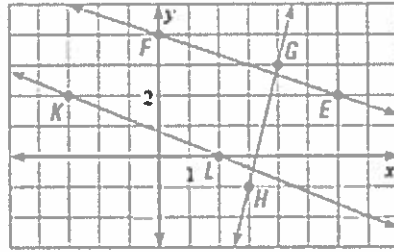
line p_2 : $3y - 5x = -33$

LINE RELATIONSHIPS Find the slope of each line. Identify any parallel or perpendicular lines.

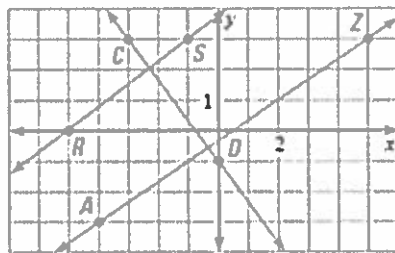
33.



34.



35.



36.

