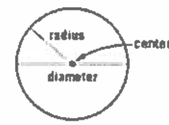
10.1 Tangents to Circles

A **circle** is the set of all points in a plane that are equidistant from a given point, called the **center** of the circle. A circle with center P is called "circle P", or $\odot P$.

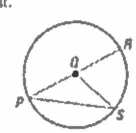
The distance from the center to a point on the circle is the **radius** of the circle. Two circles are **congruent** if they have the same radius.

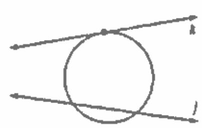
The distance across the circle, through its center, is the **diameter** of the circle. The diameter is twice the radius.



The terms *radius* and *diameter* describe segments as well as measures.

A **radius** is a segment whose endpoints are the center of the circle and a point on the circle. $\overline{QP}$, $\overline{QR}$, and $\overline{QS}$ are radii of $\odot Q$ below. All radii of a circle are congruent.





A **chord** is a segment whose endpoints are points on the circle. $\overline{PS}$ and $\overline{PR}$ are chords.

A **diameter** is a chord that passes through the center of the circle. $\overline{PR}$ is a diameter.

A **secant** is a line that intersects a circle in two points. Line j is a secant.

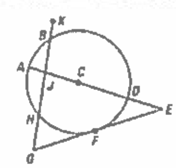
A **tangent** is a line in the plane of a circle that intersects the circle in exactly one point. Line k is a tangent.

EXAMPLE 1 ***Identifying Special Segments and Lines***

Tell whether the line or segment is best described as a *chord*, a *secant*, a *tangent*, a *diameter*, or a *radius* of $\odot C$.

a. $\overline{AD}$ b. $\overline{CD}$

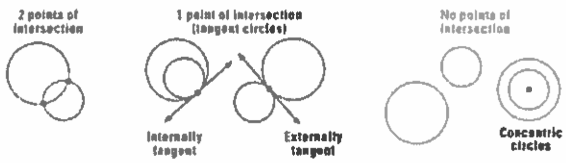
c. $\overleftrightarrow{EG}$ d. $\overline{HB}$



SOLUTION

a. $\overline{AD}$ is a diameter because it contains the center C.

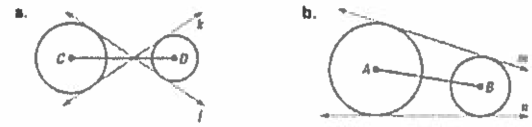
In a plane, two circles can intersect in two points, one point, or no points. Coplanar circles that intersect in one point are called **tangent circles**. Coplanar circles that have a common center are called **concentric**.



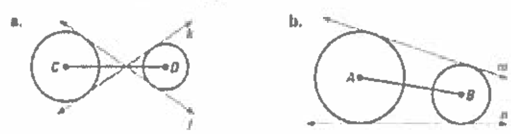
A line or segment that is tangent to two coplanar circles is called a common tangent. A *common internal tangent* intersects the segment that joins the centers of the two circles. A *common external tangent* does not intersect the segment that joins the centers of the two circles.

EXAMPLE 2 ***Identifying Common Tangents***

Tell whether the common tangents are *internal* or *external*.



Tell whether the common tangents are *internal* or *external*.



SOLUTION

a. The lines j and k intersect $\overline{CD}$, so they are common internal tangents.

b. The lines m and n do not intersect $\overline{AB}$, so they are common external tangents.

In a plane, the **interior of a circle** consists of the points that are inside the circle. The **exterior of a circle** consists of the points that are outside the circle.

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GOAL 2 Using Properties of Tangents

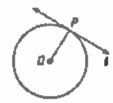
The point at which a tangent line intersects the circle to which it is tangent is the **point of tangency**. You will justify the following theorems in the exercises.

THEOREMS

THEOREM 10.1

If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.

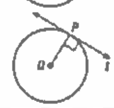
If ℓ is tangent to $\odot Q$ at P, then $\ell \perp \overline{QP}$.



THEOREM 10.2

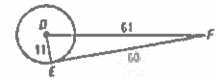
In a plane, if a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.

If $\ell \perp \overline{QP}$ at P, then ℓ is tangent to $\odot Q$.



EXAMPLE 4 *Verifying a Tangent to a Circle*

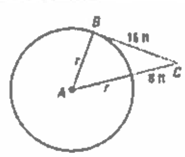
You can use the Converse of the Pythagorean Theorem to tell whether $\overleftrightarrow{EF}$ is tangent to $\odot D$.



Because $11^2 + 60^2 = 61^2$, $\triangle DEF$ is a right triangle and $\overline{DE}$ is perpendicular to $\overline{EF}$. So, by Theorem 10.2, $\overleftrightarrow{EF}$ is tangent to $\odot D$.

EXAMPLE 5 *Finding the Radius of a Circle*

You are standing at C, 8 feet from a grain silo. The distance from you to a point of tangency on the tank is 16 feet. What is the radius of the silo?

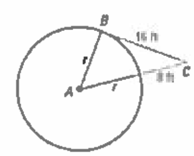


SOLUTION

Tangent $\overleftrightarrow{BC}$ is perpendicular to radius $\overline{AB}$ at B, so $\triangle ABC$ is a right triangle. So, you can use the Pythagorean Theorem.

EXAMPLE 5 *Finding the Radius of a Circle*

You are standing at C, 8 feet from a grain silo. The distance from you to a point of tangency on the tank is 16 feet. What is the radius of the silo?



SOLUTION

Tangent $\overleftrightarrow{BC}$ is perpendicular to radius $\overline{AB}$ at B, so $\triangle ABC$ is a right triangle. So, you can use the Pythagorean Theorem.

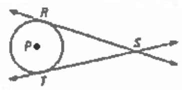
$(r + 8)^2 = r^2 + 16^2$	Pythagorean Theorem
$r^2 + 16r + 64 = r^2 + 256$	Square of binomial
$16r + 64 = 256$	Subtract r^2 from each side.
$16r = 192$	Subtract 64 from each side.
$r = 12$	Divide.

▶ The radius of the silo is 12 feet.

THEOREM

THEOREM 10.2

If two segments from the same exterior point are tangent to a circle, then they are congruent.

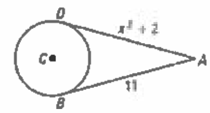


If $\overline{SR}$ and $\overline{ST}$ are tangent to $\odot P$, then $\overline{SR} \cong \overline{ST}$.

EXAMPLE 7 *Using Properties of Tangents*

$\overline{AB}$ is tangent to $\odot C$ at B.
$\overline{AD}$ is tangent to $\odot C$ at D.

Find the value of x.



EXAMPLE 7 *Using Properties of Tangents*

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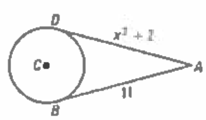


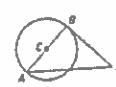
SOLUTION

$AB = AD$	Two tangent segments from the same point are $\cong$.
$11 = x^2 + 2$	Substitute.
$9 = x^2$	Subtract 2 from each side.
$\pm 3 = x$	Find the square roots of 9.

▶ The value of x is 3 or -3.

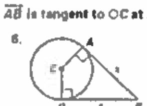
Skill Check ✓

5. In the diagram at the right, $AB = BD = 5$ and $AD = 7$. Is $\overline{BD}$ tangent to $\odot C$? Explain.

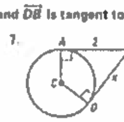


$\overline{AB}$ is tangent to $\odot C$ at A and $\overline{DB}$ is tangent to $\odot C$ at D. Find the value of x.

6.



7.



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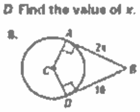


FINDING RADII The diameter of a circle is given. Find the radius.

9. $d = 15$ cm 10. $d = 6.7$ in. 11. $d = 3$ ft 12. $d = 8$ cm

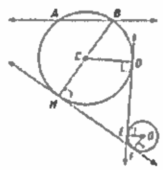
FINDING DIAMETERS The radius of $\odot C$ is given. Find the diameter of $\odot C$.

13. $r = 26$ in. 14. $r = 62$ ft 15. $r = 8.7$ in. 16. $r = 4.4$ cm

17. **CONGRUENT CIRCLES** Which two circles below are congruent? Explain your reasoning.

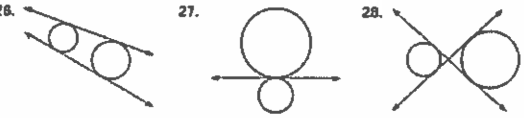
MATCHING TERMS Match the notation with the term that best describes it.

18. $\overline{AB}$ A. Center
19. H B. Chord
20. $\overleftrightarrow{HF}$ C. Diameter
21. $\overline{CH}$ D. Radius
22. C E. Point of tangency
23. $\overline{HB}$ F. Common external tangent
24. $\overleftrightarrow{AB}$ G. Common internal tangent
25. $\overleftrightarrow{DE}$ H. Secant



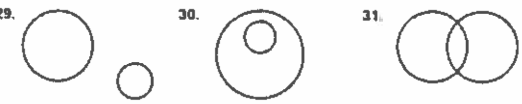
IDENTIFYING TANGENTS Tell whether the common tangent(s) are *internal* or *external*.

26. 27. 28.



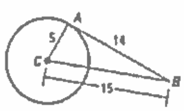
DRAWING TANGENTS Copy the diagram. Tell how many common tangents the circles have. Then sketch the tangents.

29. 30. 31.



DETERMINING TANGENCY Tell whether $\overleftrightarrow{AB}$ is tangent to $\odot C$. Explain your reasoning.

36.



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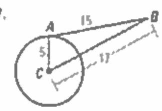

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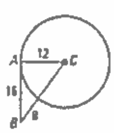

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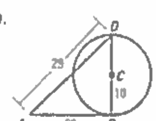
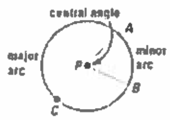


10.2 Arcs and Chords

GOAL 1 Using Arcs of Circles

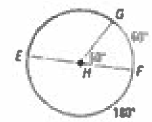
In a plane, an angle whose vertex is the center of a circle is a **central angle** of the circle.

If the measure of a central angle, $\angle APB$, is less than 180°, then A and B and the points of $\odot P$ in the interior of $\angle APB$ form a **minor arc** of the circle. The points A and B and the points of $\odot P$ in the *exterior* of $\angle APB$ form a **major arc** of the circle. If the endpoints of an arc are the endpoints of a diameter, then the arc is a **semicircle**.



NAMING ARCS Arcs are named by their endpoints. For example, the minor arc associated with $\angle APB$ above is $\overset{\frown}{AB}$. Major arcs and semicircles are named by their endpoints and by a point on the arc. For example, the major arc associated with $\angle APB$ above is $\overset{\frown}{ACB}$. $\overset{\frown}{EGF}$ below is a semicircle.

MEASURING ARCS The **measure of a minor arc** is defined to be the measure of its central angle. For instance, $m\overset{\frown}{GF} = m\angle GHF = 60°$. "$m\overset{\frown}{GF}$" is read "the measure of arc GF." You can write the measure of an arc next to the arc. The measure of a semicircle is 180°.



The **measure of a major arc** is defined as the difference between 360° and the measure of its associated minor arc. For example, $m\overset{\frown}{GEF} = 360° - 60° = 300°$. The measure of a whole circle is 360°.

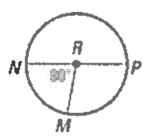
EXAMPLE 1 *Finding Measures of Arcs*

Find the measure of each arc of $\odot R$.

a. $\overset{\frown}{MN}$

b. $\overset{\frown}{MPN}$

c. $\overset{\frown}{PMN}$

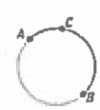


POSTULATE

POSTULATE 26 *Arc Addition Postulate*

The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

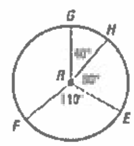
$$m\overset{\frown}{ABC} = m\overset{\frown}{AB} + m\overset{\frown}{BC}$$



EXAMPLE 2 ***Finding Measures of Arcs***

Find the measure of each arc.

a. $\widehat{GE}$ b. $\widehat{GEF}$ c. $\widehat{GF}$



SOLUTION

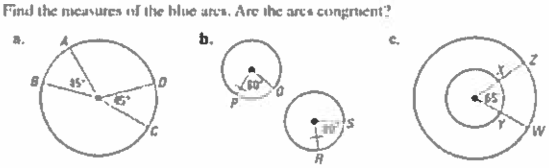
a. $m\widehat{GE} = m\widehat{GH} + m\widehat{HE} = 40° + 80° = 120°$

b. $m\widehat{GEF} = m\widehat{GE} + m\widehat{EF} = 120° + 110° = 230°$

c. $m\widehat{GF} = 360° - m\widehat{GEF} = 360° - 230° = 130°$

Two arcs of the same circle or of congruent circles are **congruent arcs** if they have the same measure. So, two minor arcs of the same circle or of congruent circles are congruent if their central angles are congruent.

Find the measures of the blue arcs. Are the arcs congruent?



SOLUTION

a. $\widehat{AB}$ and $\widehat{DC}$ are in the same circle and $m\widehat{AB} = m\widehat{DC} = 45°$. So, $\widehat{AB} \cong \widehat{DC}$.

b. $\widehat{PQ}$ and $\widehat{RS}$ are in congruent circles and $m\widehat{PQ} = m\widehat{RS} = 80°$. So, $\widehat{PQ} \cong \widehat{RS}$.

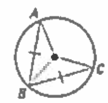
c. $m\widehat{XY} = m\widehat{ZW} = 65°$, but $\widehat{XY}$ and $\widehat{ZW}$ are not arcs of the same circle or of congruent circles, so $\widehat{XY}$ and $\widehat{ZW}$ are *not* congruent.

THEOREMS ABOUT CHORDS OF CIRCLES

THEOREM 10.4

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

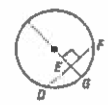
$\widehat{AB} \cong \widehat{BC}$ if and only if $\overline{AB} \cong \overline{BC}$.



THEOREM 10.5

If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.

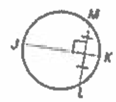
$\overline{DE} \cong \overline{EF}$, $\widehat{DG} \cong \widehat{GF}$



THEOREM 10.6

If one chord is a perpendicular bisector of another chord, then the first chord is a diameter.

$\overline{JK}$ is a diameter of the circle.



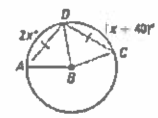
EXAMPLE 4 *Using Theorem 10.4*

You can use Theorem 10.4 to find $m\widehat{AD}$.

Because $\overline{AD} \cong \overline{DC}$, $\widehat{AD} \cong \widehat{DC}$. So, $m\widehat{AD} = m\widehat{DC}$.

$2x = x + 40$ **Substitute.**

$x = 40$ **Subtract x from each side.**

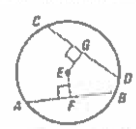


THEOREM

THEOREM 10.7

In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

$\overline{AB} \cong \overline{CD}$ if and only if $\overline{EF} \cong \overline{EG}$.

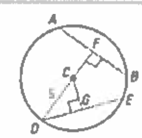


EXAMPLE 7 *Using Theorem 10.7*

$AB = 8$, $DE = 8$, and $CD = 5$. Find CF.

SOLUTION

Because $\overline{AB}$ and $\overline{DE}$ are congruent chords, they are equidistant from the center. So, $\overline{CF} \cong \overline{CG}$. To find CG, first find DG.



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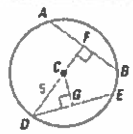


SOLUTION

Because $\overline{AB}$ and $\overline{DE}$ are congruent chords, they are equidistant from the center. So, $\overline{CF} \cong \overline{CG}$. To find CG, first find DG.

$\overline{CG} \perp \overline{DE}$, so $\overline{CG}$ bisects $\overline{DE}$. Because $DE = 8$, $DG = \frac{8}{2} = 4$.

Then use DG to find CG.

$DG = 4$ and $CD = 5$, so $\triangle CGD$ is a 3-4-5 right triangle. So, $CG = 3$.

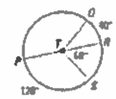
Finally, use CG to find CF.

▶ Because $\overline{CF} \cong \overline{CG}$, $CF = CG = 3$.

Skill Check ✓

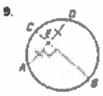
Find the measure in ⊙T.

3. $m\overset{\frown}{RS}$
4. $m\overset{\frown}{RPS}$
5. $m\overset{\frown}{PQR}$
6. $m\overset{\frown}{QS}$
7. $m\overset{\frown}{QSP}$
8. $m\angle QTR$



What can you conclude about the diagram? State a postulate or theorem that justifies your answer.

9.



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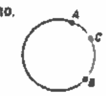
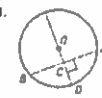
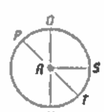


11.



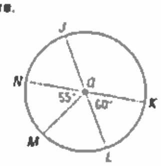
UNDERSTANDING THE CONCEPT **Determine whether the arc is a *minor arc*, a *major arc*, or a *semicircle* of ⊙R.**

12. $\overset{\frown}{PQ}$
13. $\overset{\frown}{SU}$
14. $\overset{\frown}{PQT}$
15. $\overset{\frown}{QT}$
16. $\overset{\frown}{TUQ}$
17. $\overset{\frown}{TUP}$
18. $\overset{\frown}{QUT}$
19. $\overset{\frown}{PUQ}$



MEASURING ARCS AND CENTRAL ANGLES **$\overline{KN}$ and $\overline{JL}$ are diameters. Copy the diagram. Find the indicated measure.**

20. $m\overset{\frown}{KL}$
21. $m\overset{\frown}{MN}$
22. $m\overset{\frown}{LNK}$
23. $m\overset{\frown}{MKN}$
24. $m\overset{\frown}{NJK}$
25. $m\overset{\frown}{JML}$
26. $m\angle JQN$
27. $m\angle MQL$
28. $m\overset{\frown}{JN}$
29. $m\overset{\frown}{ML}$
30. $m\overset{\frown}{JM}$
31. $m\overset{\frown}{LN}$

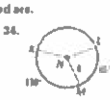


FINDING ARC MEASURES **Find the measure of the red arc.**

32.

33.

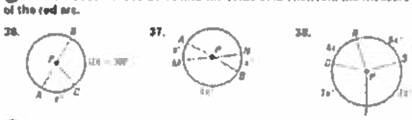
34.



35. Name two pairs of congruent arcs in Exercises 32–34. Explain your reasoning.

USING ALGEBRA **Use ⊙P to find the value of x. Then find the measure of the red arc.**

36.



37.

38.

LOGICAL REASONING **What can you conclude about the diagram? State a postulate or theorem that justifies your answer.**

39.

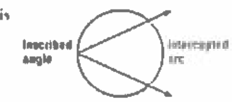
40.

41.

10.3 Inscribed Angles

GOAL 1 USING INSCRIBED ANGLES

An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. The arc that lies in the interior of an inscribed angle and has endpoints on the angle is called the **intercepted arc** of the angle.

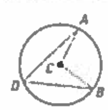


THEOREM

THEOREM 10.8 *Measure of an Inscribed Angle*

If an angle is inscribed in a circle, then its measure is half the measure of its intercepted arc.

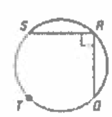
$m\angle ADB = \frac{1}{2}m\overset{\frown}{AB}$



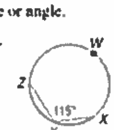
EXAMPLE 1 *Finding Measures of Arcs and Inscribed Angles*

Find the measure of the blue arc or angle.

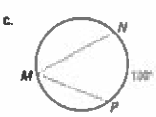
a.



b.

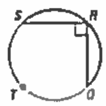


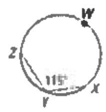
c.

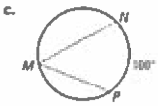


EXAMPLE 1 ***Finding Measures of Arcs and Inscribed Angles***

Find the measure of the blue arc or angle.

a. 

b. 

c. 

SOLUTION

a. $m\widehat{QTS} = 2m\angle QRS = 2(90°) = 180°$

b. $m\widehat{ZWX} = 2m\angle ZYX = 2(115°) = 230°$

c. $m\angle NMP = \frac{1}{2}m\widehat{NP} = \frac{1}{2}(100°) = 50°$

THEOREM

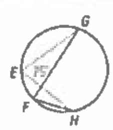
THEOREM 10.9

If two inscribed angles of a circle intercept the same arc, then the angles are congruent.

$\angle C \cong \angle D$

EXAMPLE 3 ***Finding the Measure of an Angle***

It is given that $m\angle E = 75°$. What is $m\angle F$?



SOLUTION

$\angle E$ and $\angle F$ both intercept $\widehat{GH}$, so $\angle E \cong \angle F$.

▶ So, $m\angle F = m\angle E = 75°$.

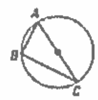
GOAL 2 **USING PROPERTIES OF INSCRIBED POLYGONS**

If all of the vertices of a polygon lie on a circle, the polygon is **inscribed** in the circle and the circle is **circumscribed** about the polygon. The polygon is an *inscribed polygon* and the circle is a *circumscribed circle*. You are asked to justify Theorem 10.10 and part of Theorem 10.11 in Exercises 39 and 40. A complete proof of Theorem 10.11 appears on page 840.

THEOREMS ABOUT INSCRIBED POLYGONS

THEOREM 10.10

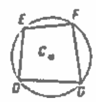
If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.



$\angle B$ is a right angle if and only if AC is a diameter of the circle.

THEOREM 10.11

A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.

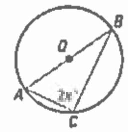


D, E, F, and G lie on some circle, $\odot C$, if and only if $m\angle D + m\angle F = 180°$ and $m\angle E + m\angle G = 180°$.

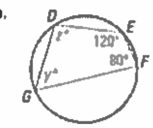
EXAMPLE 5 *Using Theorems 10.10 and 10.11*

Find the value of each variable.

a.



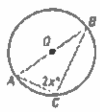
b.



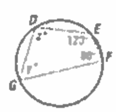
EXAMPLE 5 *Using Theorems 10.10 and 10.11*

Find the value of each variable.

a.



b.



SOLUTION

a. $\overline{AB}$ is a diameter. So, $\angle C$ is a right angle and $m\angle C = 90°$.

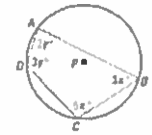
$$2x° = 90°$$
$$x = 45$$

b. $DEFG$ is inscribed in a circle, so opposite angles are supplementary.

$$m\angle D + m\angle F = 180° \qquad m\angle E + m\angle G = 180°$$
$$z + 80 = 180 \qquad 120 + y = 180$$
$$z = 100 \qquad y = 60$$

EXAMPLE 6 *Using an Inscribed Quadrilateral*

In the diagram, $ABCD$ is inscribed in $\odot P$. Find the measure of each angle.



SOLUTION

$ABCD$ is inscribed in a circle, so opposite angles are supplementary.

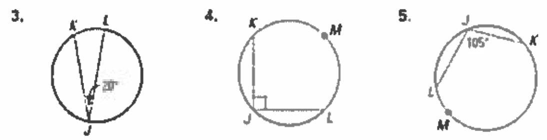
$$3x + 3y = 180 \qquad 5x + 2y = 180$$

To solve this system of linear equations, you can solve the first equation for y to get $y = 60 - x$. Substitute this expression into the second equation.

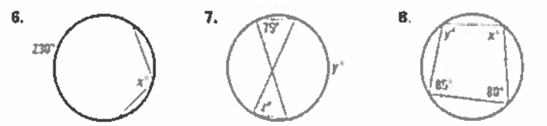
$5x + 2y = 180$	Write second equation.
$5x + 2(60 - x) = 180$	Substitute $60 - x$ for y.
$5x + 120 - 2x = 180$	Distributive property
$3x = 60$	Subtract 120 from each side.
$x = 20$	Divide each side by 3.
$y = 60 - 20 = 40$	Substitute and solve for y.

▶ $x = 20$ and $y = 40$, so $m\angle A = 80°$, $m\angle B = 60°$, $m\angle C = 100°$, and $m\angle D = 120°$.

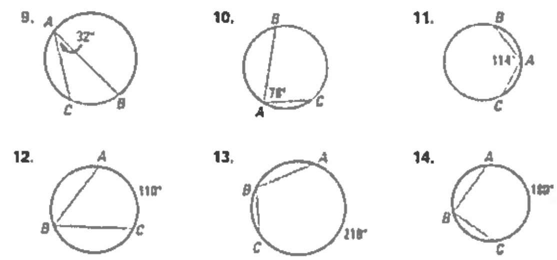
Find the measure of the blue arc.



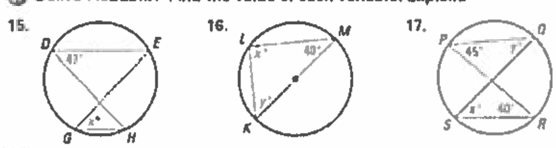
Find the value of each variable.



ARC AND ANGLE MEASURES **Find the measure of the blue arc or angle.**

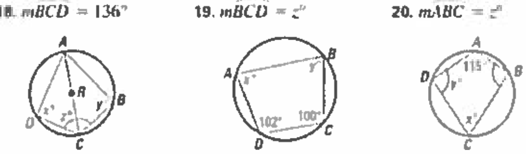


USING ALGEBRA **Find the value of each variable. Explain.**

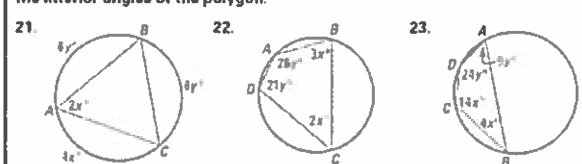


USING ALGEBRA **Find the values of *x*, *y*, and *z*.**

18. $m\widehat{BCD} = 136°$ **19.** $m\widehat{BCD} = z°$ **20.** $m\widehat{ABC} = z°$



USING ALGEBRA **Find the values of *x* and *y*. Then find the measures of the interior angles of the polygon.**



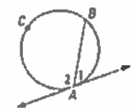
10.4 Other Angle Relationships in Circles

THEOREM

THEOREM 10.12

If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one half the measure of its intercepted arc.

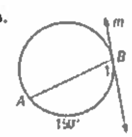
$m\angle 1 = \frac{1}{2} m\overset{\frown}{AB}$ $\qquad$ $m\angle 2 = \frac{1}{2} m\overset{\frown}{BCA}$



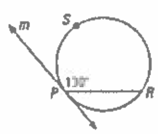
EXAMPLE 1 ***Finding Angle and Arc Measures***

Line m is tangent to the circle. Find the measure of the red angle or arc.

a.

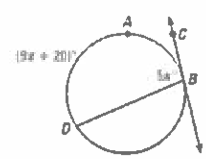


b.



EXAMPLE 2 ***Finding an Angle Measure***

In the diagram below, $\overleftrightarrow{BC}$ is tangent to the circle. Find $m\angle CBD$.



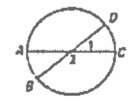
If two lines intersect a circle, there are three places where the lines can intersect.

You know how to find angle and arc measures when lines intersect *on* the circle. You can use Theorems 10.13 and 10.14 to find measures when the lines intersect *inside* or *outside* the circle. You will prove these theorems in Exercises 40 and 41.

THEOREMS

THEOREM 10.13

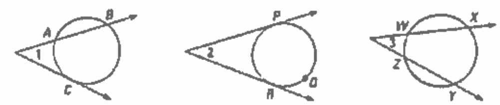
If two chords intersect in the *interior* of a circle, then the measure of each angle is one half the *sum* of the measures of the arcs intercepted by the angle and its vertical angle.



$m\angle 1 = \frac{1}{2}(m\overset{\frown}{CD} + m\overset{\frown}{AB}),\ m\angle 2 = \frac{1}{2}(m\overset{\frown}{BC} + m\overset{\frown}{AD})$

THEOREM 10.14

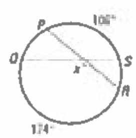
If a tangent and a secant, two tangents, or two secants intersect in the *exterior* of a circle, then the measure of the angle formed is one half the *difference* of the measures of the intercepted arcs.



$m\angle 1 = \frac{1}{2}(m\overset{\frown}{BC} - m\overset{\frown}{AC})$ $m\angle 2 = \frac{1}{2}(m\overset{\frown}{PQR} - m\overset{\frown}{PR})$ $m\angle 3 = \frac{1}{2}(m\overset{\frown}{XY} - m\overset{\frown}{WZ})$

EXAMPLE 3 ***Finding the Measure of an Angle Formed by Two Chords***

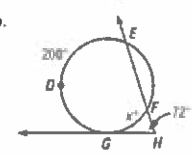
Find the value of x.



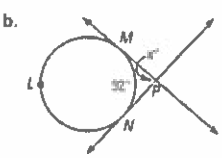
EXAMPLE 4 ***Using Theorem 10.14***

Find the value of x.

a.



b.



SOLUTION

a. $m\angle GHF = \frac{1}{2}(m\overparen{EDG} - m\overparen{GF})$ **Apply Theorem 10.14.**

$72° = \frac{1}{2}(200° - x°)$ **Substitute.**

$144 = 200 - x$ **Multiply each side by 2.**

$x = 56$ **Solve for *x*.**

b. Because $\overparen{MN}$ and $\overparen{MLN}$ make a whole circle, $m\overparen{MLN} = 360° - 92° = 268°$.

$x = \frac{1}{2}(m\overparen{MLN} - m\overparen{MN})$ **Apply Theorem 10.14.**

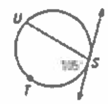
$= \frac{1}{2}(268 - 92)$ **Substitute.**

$= \frac{1}{2}(176)$ **Subtract.**

$= 88$ **Multiply.**

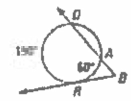
Find the indicated measure or value.

2. $m\overparen{STU}$

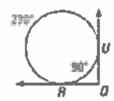


3. $m\angle 1$

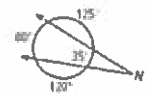
4. $m\angle DBR$



5. $m\angle RQU$



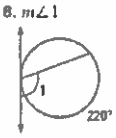
6. $m\angle N$



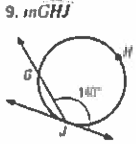
7. $m\angle 1$

FINDING MEASURES **Find the indicated measure.**

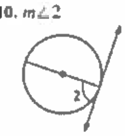
8. $m\angle 1$



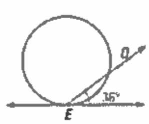
9. $m\overparen{GHJ}$



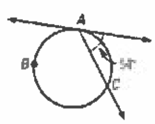
10. $m\angle 2$



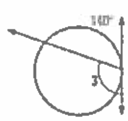
11. $m\overparen{DE}$



12. $m\overparen{ABC}$

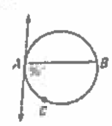


13. $m\angle 3$

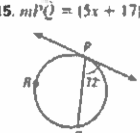


USING ALGEBRA **Find the value of *x*.**

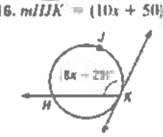
14. $m\overparen{AB} = x°$

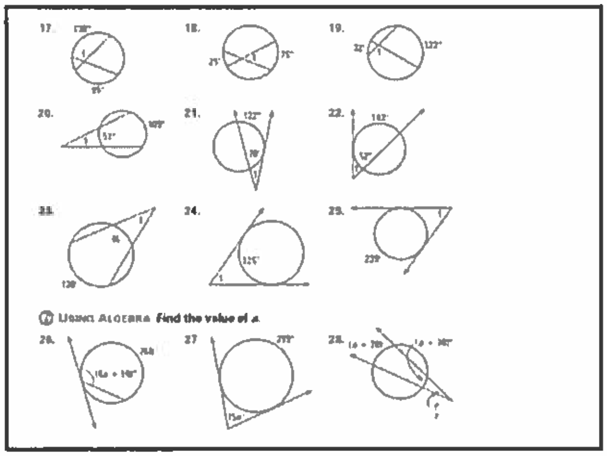


15. $m\overparen{PQ} = (5x + 17)°$



16. $m\overparen{HJK} = (10x + 50)°$



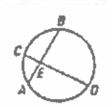


10.5 Segment Lengths in Circles

THEOREM

THEOREM 10.15

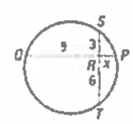
If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.



$EA \cdot EB = EC \cdot ED$

EXAMPLE 1 *Finding Segment Lengths*

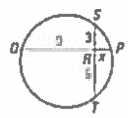
Chords $\overline{ST}$ and $\overline{PQ}$ intersect inside the circle. Find the value of x.



SOLUTION

EXAMPLE 1 ***Finding Segment Lengths***

Chords $\overline{ST}$ and $\overline{PQ}$ intersect inside the circle. Find the value of x.

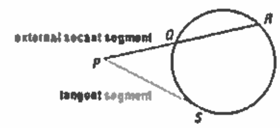


SOLUTION

$RQ \cdot RP = RS \cdot RT$	Use Theorem 10.15.
$9 \cdot x = 3 \cdot 6$	Substitute.
$9x = 18$	Simplify.
$x = 2$	Divide each side by 9.

GOAL 2 **USING SEGMENTS OF TANGENTS AND SECANTS**

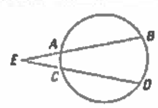
In the figure shown below, $\overline{PS}$ is called a **tangent segment** because it is tangent to the circle at an endpoint. Similarly, $\overline{PR}$ is a **secant segment** and $\overline{PQ}$ is the **external segment** of $\overline{PR}$.



THEOREMS

THEOREM 10.16

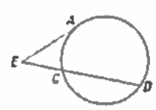
If two secant segments share the same endpoint outside a circle, then the product of the length of one secant segment and the length of its external segment equals the product of the length of the other secant segment and the length of its external segment.



$EA \cdot EB = EC \cdot ED$

THEOREM 10.17

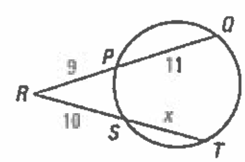
If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the length of the secant segment and the length of its external segment equals the square of the length of the tangent segment.



$(EA)^2 = EC \cdot ED$

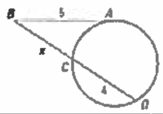
EXAMPLE 2 ***Finding Segment Lengths***

Find the value of x.



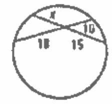
EXAMPLE 4 ***Finding Segment Lengths***

Use the figure at the right to find the value of x.

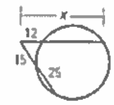


Fill in the blanks. Then find the value of x.

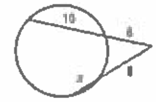
3. $x \cdot \underline{?} = 10 \cdot \underline{?}$



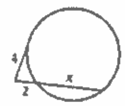
4. $\underline{?} \cdot x = \underline{?} \cdot 40$



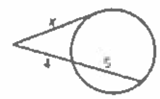
5. $6 \cdot \underline{?} = 8 \cdot \underline{?}$



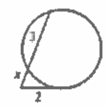
6. $4^2 = 2 \cdot (\underline{?} + x)$



7. $x^2 = 4 \cdot \underline{?}$

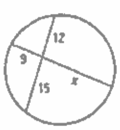


8. $x \cdot \underline{?} = \underline{?}$

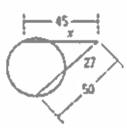


FINDING SEGMENT LENGTHS **Fill in the blanks. Then find the value of x.**

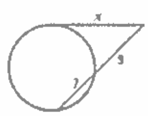
10. $x \cdot \underline{?} = 12 \cdot \underline{?}$



11. $x \cdot \underline{?} = \underline{?} \cdot 50$

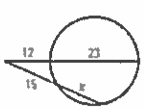


12. $x^2 = 9 \cdot \underline{?}$



FINDING SEGMENT LENGTHS **Find the value of x.**

13.



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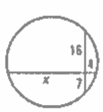

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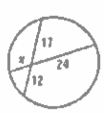
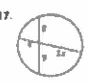


FINDING SEGMENT LENGTHS **Find the value of x.**

16.

17.



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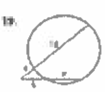

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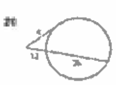

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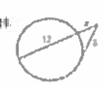

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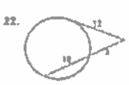

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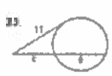

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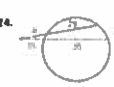
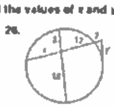


USING ALGEBRA **Find the values of x and y.**

25.

26.

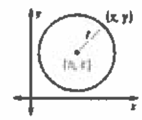


27.

10.6 Equations of Circles

GOAL 1 **FINDING EQUATIONS OF CIRCLES**

You can write an equation of a circle in a coordinate plane if you know its radius and the coordinates of its center. Suppose the radius of a circle is r and the center is (h, k). Let (x, y) be any point on the circle. The distance between (x, y) and (h, k) is r, so you can use the Distance Formula.



$$\sqrt{(x-h)^2 + (y-k)^2} = r$$

Square both sides to find the **standard equation of a circle** with radius r and center (h, k).

Standard equation of a circle: $(x-h)^2 + (y-k)^2 = r^2$

If the center is the origin, then the standard equation is $x^2 + y^2 = r^2$.

EXAMPLE 1 ***Writing a Standard Equation of a Circle***

Write the standard equation of the circle with center $(-4, 0)$ and radius 7.1.

EXAMPLE 1 ***Writing a Standard Equation of a Circle***

Write the standard equation of the circle with center $(-4, 0)$ and radius 7.1.

SOLUTION

$(x-h)^2 + (y-k)^2 = r^2$	Standard equation of a circle
$[x-(-4)]^2 + (y-0)^2 = 7.1^2$	Substitute.
$(x+4)^2 + y^2 = 50.41$	Simplify.

EXAMPLE 2 ***Writing a Standard Equation of a Circle***

The point (1, 2) is on a circle whose center is (5, −1). Write the standard equation of the circle.

EXAMPLE 2 ***Writing a Standard Equation of a Circle***

The point (1, 2) is on a circle whose center is (5, −1). Write the standard equation of the circle.

SOLUTION

Find the radius. The radius is the distance from the point (1, 2) to the center (5, −1).

$r = \sqrt{(5-1)^2 + (-1-2)^2}$ Use the Distance Formula.

$r = \sqrt{4^2 + (-3)^2}$ Simplify.

$r = 5$ Simplify.

Substitute $(h, k) = (5, -1)$ and $r = 5$ into the standard equation of a circle.

$(x-5)^2 + (y-(-1))^2 = 5^2$ Standard equation of a circle

$(x-5)^2 + (y+1)^2 = 25$ Simplify.

EXAMPLE 3 ***Graphing a Circle***

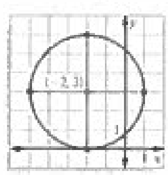
The equation of a circle is $(x+2)^2 + (y-3)^2 = 9$. Graph the circle.

Rewrite the equation to find the center and radius:

$$(x+2)^2 + (y-3)^2 = 9$$

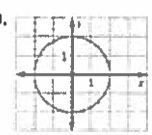
$$[x-(-2)]^2 + (y-3)^2 = 3^2$$

The center is (−2, 3) and the radius is 3. To graph the circle, place the point of a compass at (−2, 3), set the radius at 3 units, and swing the compass to draw a full circle.



Give the coordinates of the center and the radius. Write an equation of the circle in standard form.

3.



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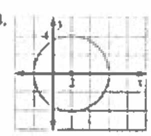

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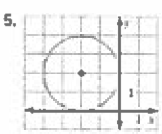

6. $P(-1, 3)$ is on a circle whose center is $C(0, 0)$. Write an equation of $\odot C$.

USING STANDARD EQUATIONS **Give the center and radius of the circle.**

7. $(x-4)^2+(y-3)^2=16$
8. $(x-5)^2+(y-1)^2=25$
9. $x^2+y^2=4$
10. $(x+2)^2+(y-3)^2=36$
11. $(x+5)^2+(y+3)^2=1$
12. $\left(x-\frac{1}{2}\right)^2+\left(y+\frac{3}{4}\right)^2=\frac{1}{4}$

USING GRAPHS **Give the coordinates of the center, the radius, and the equation of the circle.**

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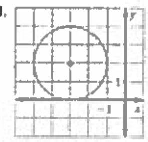

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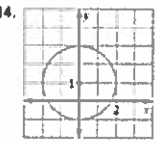

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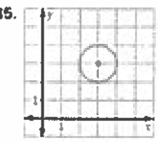

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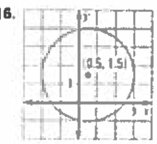

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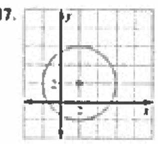

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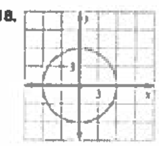

WRITING EQUATIONS **Write the standard equation of the circle with the given center and radius.**

19. center $(0, 0)$, radius 1
20. center $(4, 0)$, radius 4
21. center $(3, -2)$, radius 2
22. center $(-1, -3)$, radius 6