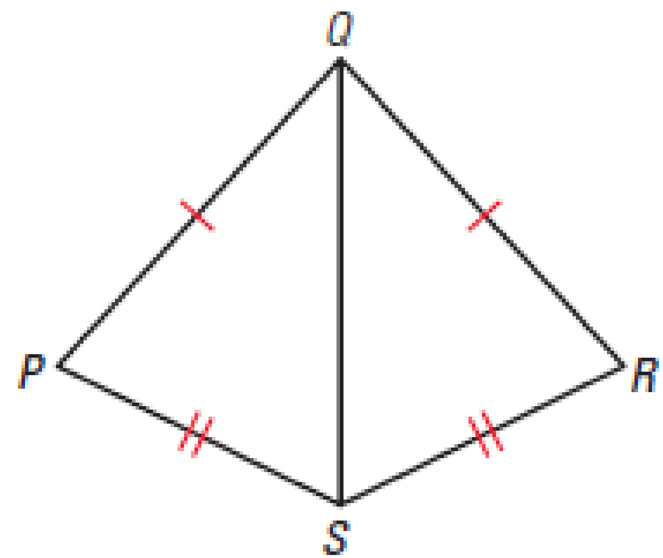
4.5 Using Congruent Triangles

Knowing that all pairs of corresponding parts of congruent triangles are congruent can help you reach conclusions about congruent figures.

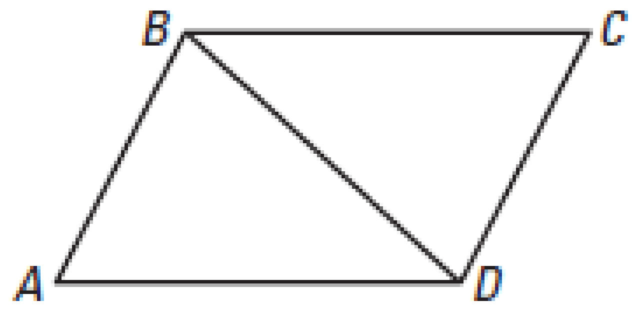
For instance, suppose you want to prove that $\angle PQS \cong \angle RQS$ in the diagram shown at the right. One way to do this is to show that $\triangle PQS \cong \triangle RQS$ by the SSS Congruence Postulate. Then you can use the fact that corresponding parts of congruent triangles are congruent to conclude that $\angle PQS \cong \angle RQS$.



EXAMPLE 1 Planning and Writing a Proof

GIVEN ▶ $\overline{AB} \parallel \overline{CD}$, $\overline{BC} \parallel \overline{DA}$

PROVE ▶ $\overline{AB} \cong \overline{CD}$

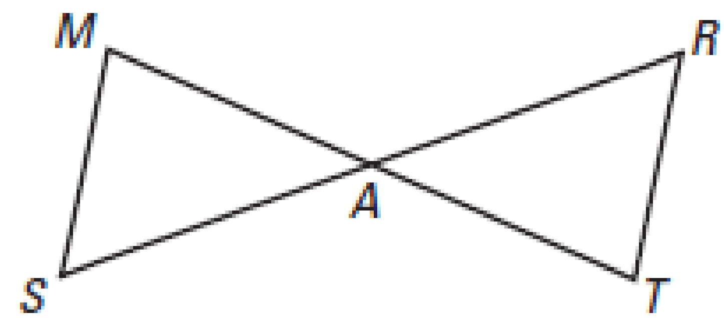


Plan for Proof Show that $\triangle ABD \cong \triangle CDB$. Then use the fact that corresponding parts of congruent triangles are congruent.

EXAMPLE 2 *Planning and Writing a Proof*

GIVEN ▶ A is the midpoint of $\overline{MT}$,
A is the midpoint of $\overline{SR}$.

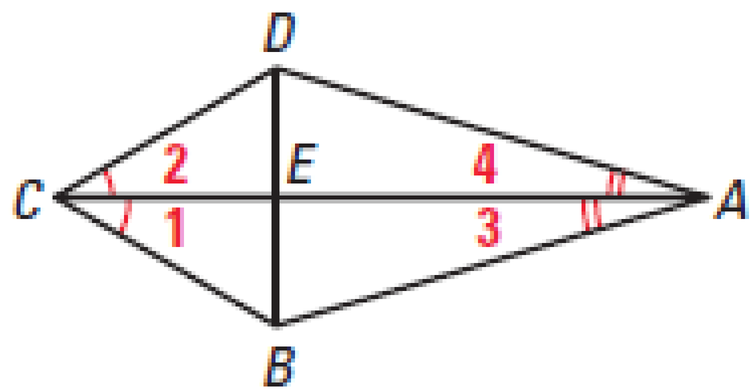
PROVE ▶ $\overline{MS} \parallel \overline{TR}$



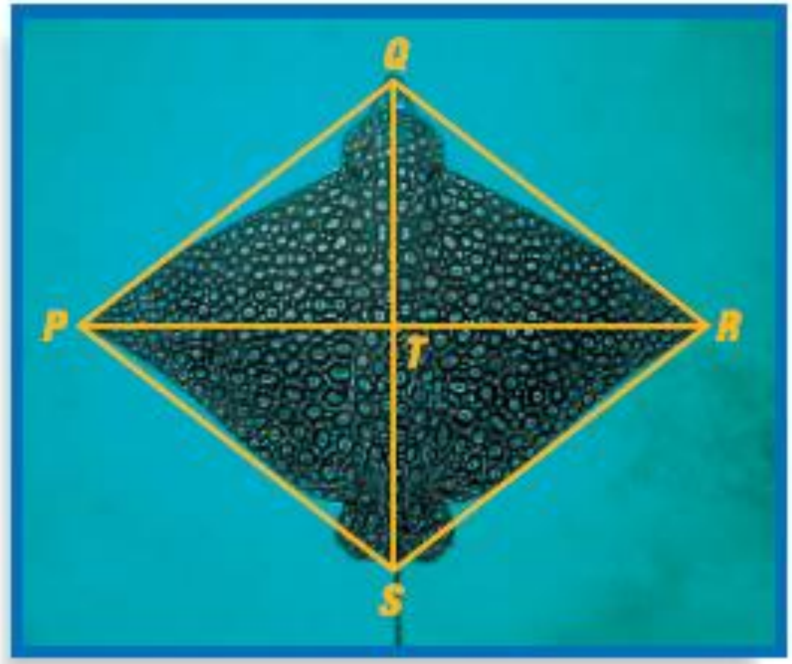
EXAMPLE 3 Using More than One Pair of Triangles

GIVEN ▶ $\angle 1 \cong \angle 2$
$\angle 3 \cong \angle 4$

PROVE ▶ $\triangle BCE \cong \triangle DCE$



In Exercises 1–3, use the photo of the eagle ray.



1. To prove that $\angle PQT \cong \angle RQT$, which triangles might you prove to be congruent?

2. If you know that the opposite sides of figure $PQRS$ are parallel, can you prove that $\triangle PQT \cong \triangle RST$? Explain.

3. The statements listed below are not in order. Use the photo to order them as statements in a two-column proof. Write a reason for each statement.

GIVEN ▸ $\overline{QS} \perp \overline{RP}$, $\overline{PT} \cong \overline{RT}$

PROVE ▸ $\overline{PS} \cong \overline{RS}$

A. $\overline{QS} \perp \overline{RP}$

B. $\triangle PTS \cong \triangle RTS$

C. $\angle PTS \cong \angle RTS$

D. $\overline{PS} \cong \overline{RS}$

E. $\overline{PT} \cong \overline{RT}$

F. $\overline{TS} \cong \overline{TS}$

G. $\angle PTS$ and $\angle RTS$ are right angles.

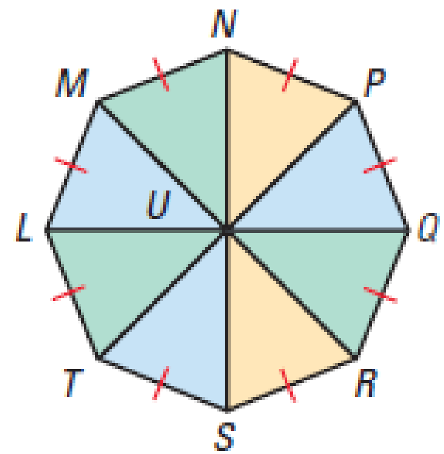
STAINED GLASS WINDOW The eight window panes in the diagram are isosceles triangles. The bases of the eight triangles are congruent.

4. Explain how you know that $\triangle NUP \cong \triangle PUQ$.

5. Explain how you know that $\triangle NUP \cong \triangle QUR$.

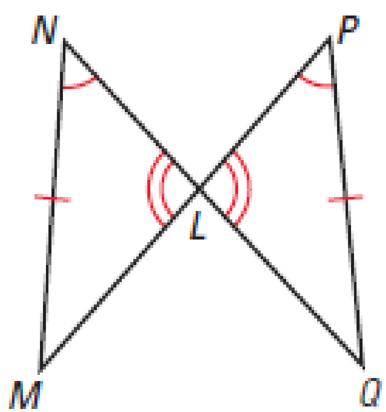
6. Do you have enough information to prove that all the triangles are congruent? Explain.

7. Explain how you know that $\angle UNP \cong \angle UPQ$.

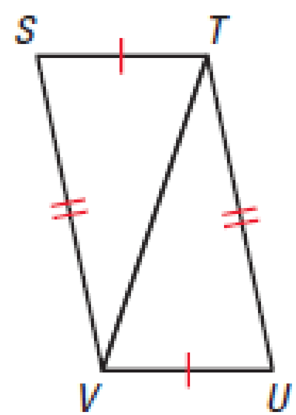


DEVELOPING PROOF State which postulate or theorem you can use to prove that the triangles are congruent. Then explain how proving that the triangles are congruent proves the given statement.

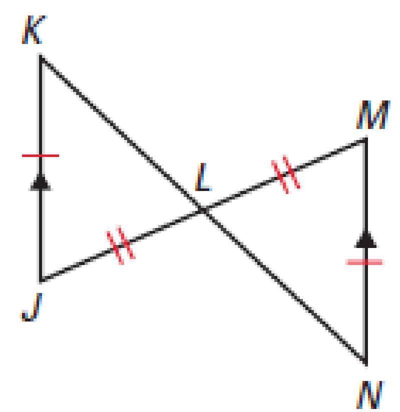
8. PROVE ▶ $\overline{ML} \cong \overline{QL}$



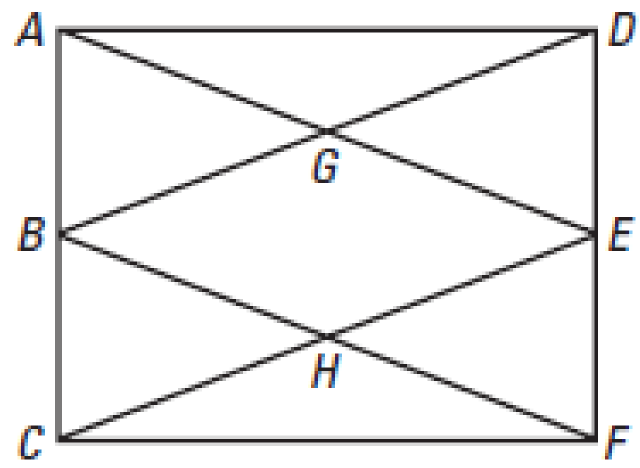
9. PROVE ▶ $\angle STV \cong \angle UVT$



10. PROVE ▶ $KL = NL$



CAT'S CRADLE Use the diagram of the string game Cat's Cradle and the information given below.



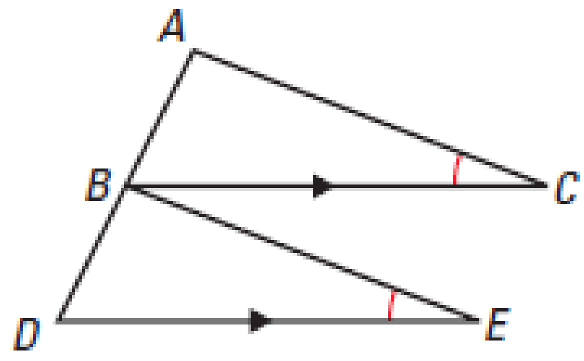
GIVEN ▶ $\triangle EDA \cong \triangle BCF$
$\triangle AGD \cong \triangle FHC$
$\triangle BFC \cong \triangle ECF$

11. PROVE ▶ $\overline{GD} \cong \overline{HC}$

12. PROVE ▶ $\angle CBH \cong \angle FEH$

13. PROVE ▶ $\overline{AE} \cong \overline{FB}$

14. **DEVELOPING PROOF** Complete the proof that $\angle BAC \cong \angle DBE$.

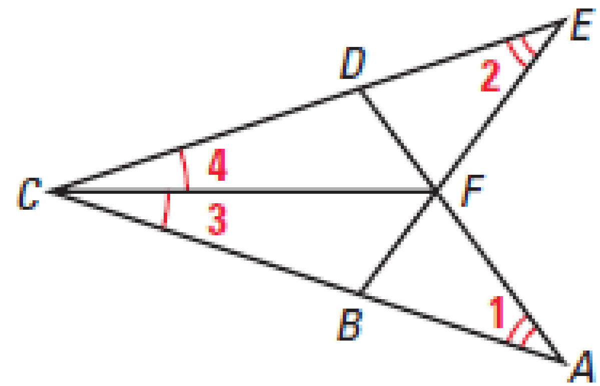


GIVEN ▶ B is the midpoint of $\overline{AD}$, $\angle C \cong \angle E$, $\overline{BC} \parallel \overline{DE}$

PROVE ▶ $\angle BAC \cong \angle DBE$

Statements	Reasons
1. B is the midpoint of $\overline{AD}$.	**1.** Given
2. $\overline{AB} \cong \overline{BD}$	**2.** ___?___
3. $\angle C \cong \angle E$	**3.** Given
4. $\overline{BC} \parallel \overline{DE}$	**4.** Given
5. $\angle EDB \cong \angle CBA$	**5.** ___?___
6. ___?___	**6.** AAS Congruence Theorem
7. $\angle BAC \cong \angle DBE$	**7.** ___?___

15. **DEVELOPING PROOF** Complete the proof that $\triangle AFB \cong \triangle EFD$.



GIVEN ▶ $\angle 1 \cong \angle 2$
$\angle 3 \cong \angle 4$

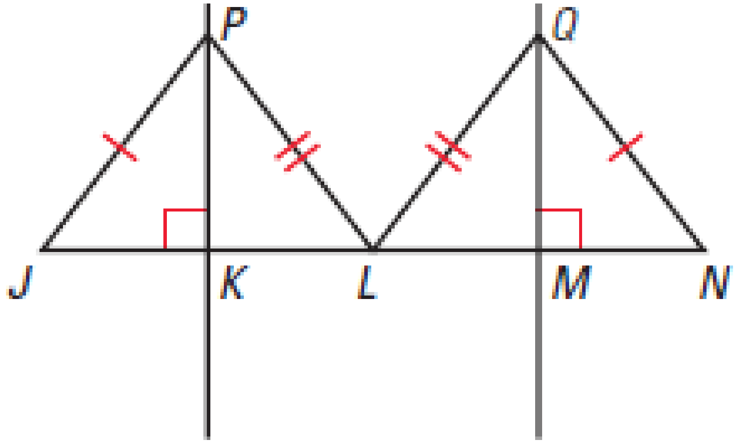
PROVE ▶ $\angle AFB \cong \angle EFD$

Statements	Reasons
1. $\angle 1 \cong \angle 2$	**1.** ___?___
2. $\angle 3 \cong \angle 4$	**2.** ___?___
3. ___?___	**3.** Reflexive Property of Congruence
4. $\triangle AFC \cong \triangle EFC$	**4.** ___?___
5. $\overline{AF} \cong \overline{EF}$	**5.** ___?___
6. ___?___	**6.** Vertical Angles Theorem
7. $\triangle AFB \cong \triangle EFD$	**7.** ___?___

16. **BRIDGES** The diagram represents a section of the framework of the Kap Shui Mun Bridge shown in the photo on page 229. Write a two-column proof to show that $\triangle PKJ \cong \triangle QMN$.

GIVEN ▶ L is the midpoint of $\overline{JN}$, $\overline{PJ} \cong \overline{QN}$, $\overline{PL} \cong \overline{QL}$, $\angle PKJ$ and $\angle QMN$ are right angles.

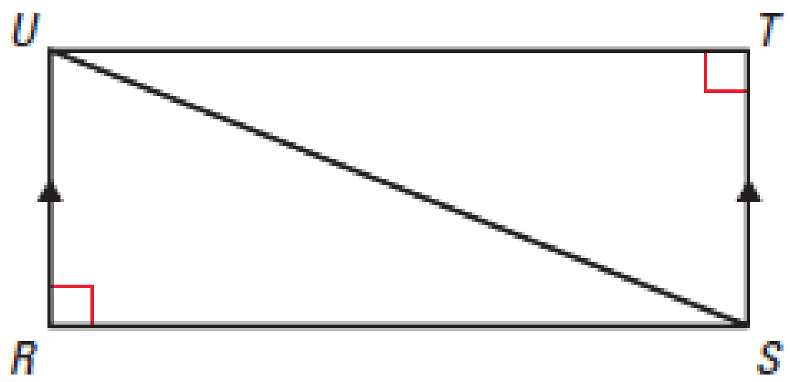
PROVE ▶ $\triangle PKJ \cong \triangle QMN$



PROOF Write a two-column proof or a paragraph proof.

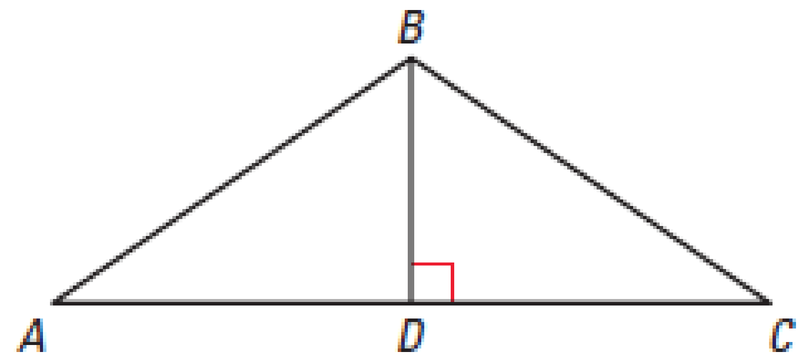
17. **GIVEN** ▶ $\overline{UR} \parallel \overline{ST}$, $\angle R$ and $\angle T$ are right angles.

PROVE ▶ $\angle RSU \cong \angle TUS$



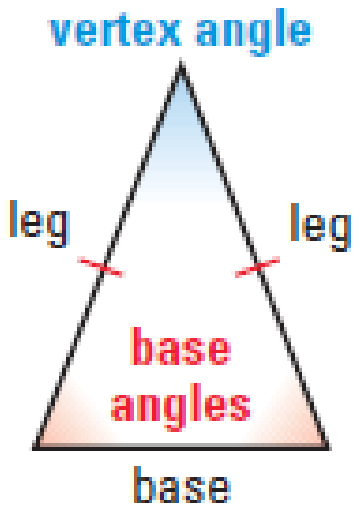
18. **GIVEN** ▶ $\overline{BD} \perp \overline{AC}$, $\overline{BD}$ bisects $\overline{AC}$.

PROVE ▶ $\angle ABD$ and $\angle BCD$ are complementary angles.



4.6 Isosceles, Equilateral, and Right Triangles

In Lesson 4.1, you learned that a triangle is isosceles if it has at least two congruent sides. If it has exactly two congruent sides, then they are the legs of the triangle and the noncongruent side is the base. The two angles adjacent to the base are the **base angles.** The angle opposite the base is the **vertex angle.**

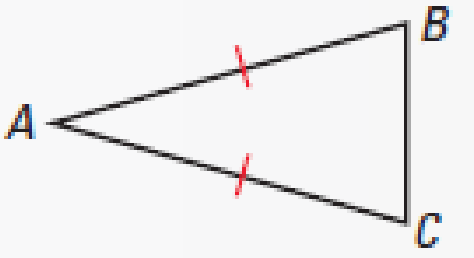


THEOREMS

THEOREM 4.6 ***Base Angles Theorem***

If two sides of a triangle are congruent, then the angles opposite them are congruent.

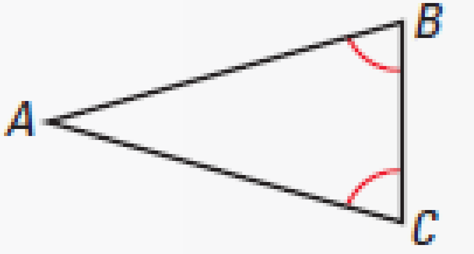
If $\overline{AB} \cong \overline{AC}$, then $\angle B \cong \angle C$.



THEOREM 4.7 ***Converse of the Base Angles Theorem***

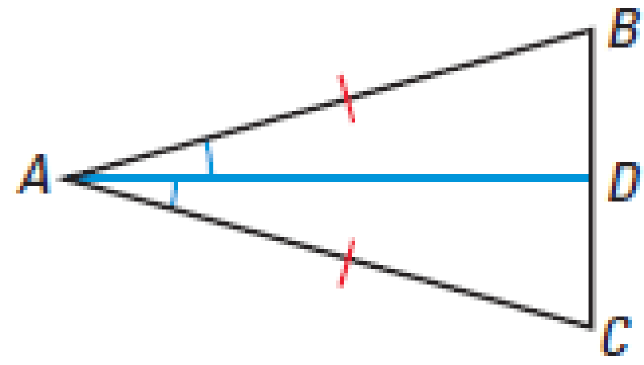
If two angles of a triangle are congruent, then the sides opposite them are congruent.

If $\angle B \cong \angle C$, then $\overline{AB} \cong \overline{AC}$.



EXAMPLE 1 Proof of the Base Angles Theorem

Use the diagram of $\triangle ABC$ to prove the Base Angles Theorem.



GIVEN ▶ $\triangle ABC$, $\overline{AB} \cong \overline{AC}$

PROVE ▶ $\angle B \cong \angle C$

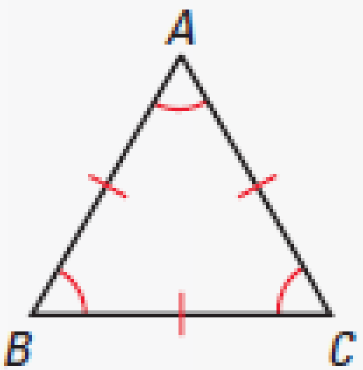
COROLLARIES

COROLLARY TO THEOREM 4.6

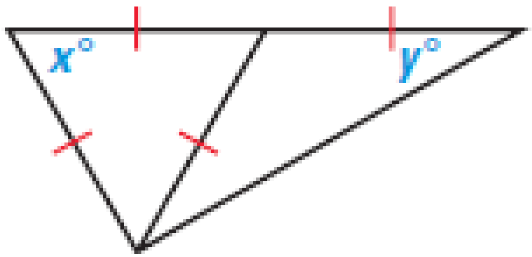
If a triangle is equilateral, then it is equiangular.

COROLLARY TO THEOREM 4.7

If a triangle is equiangular, then it is equilateral.



EXAMPLE 2 Using Equilateral and Isosceles Triangles



a. Find the value of x.

b. Find the value of y.

GOAL 2 USING PROPERTIES OF RIGHT TRIANGLES

You have learned four ways to prove that triangles are congruent.

- Side-Side-Side (SSS) Congruence Postulate (p. 212)
- Side-Angle-Side (SAS) Congruence Postulate (p. 213)
- Angle-Side-Angle (ASA) Congruence Postulate (p. 220)
- Angle-Angle-Side (AAS) Congruence Theorem (p. 220)

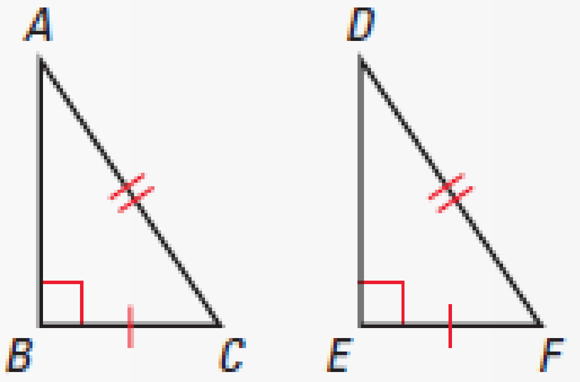
The Hypotenuse-Leg Congruence Theorem below can be used to prove that two *right* triangles are congruent. A proof of this theorem appears on page 837.

THEOREM

THEOREM 4.8 ***Hypotenuse-Leg (HL) Congruence Theorem***

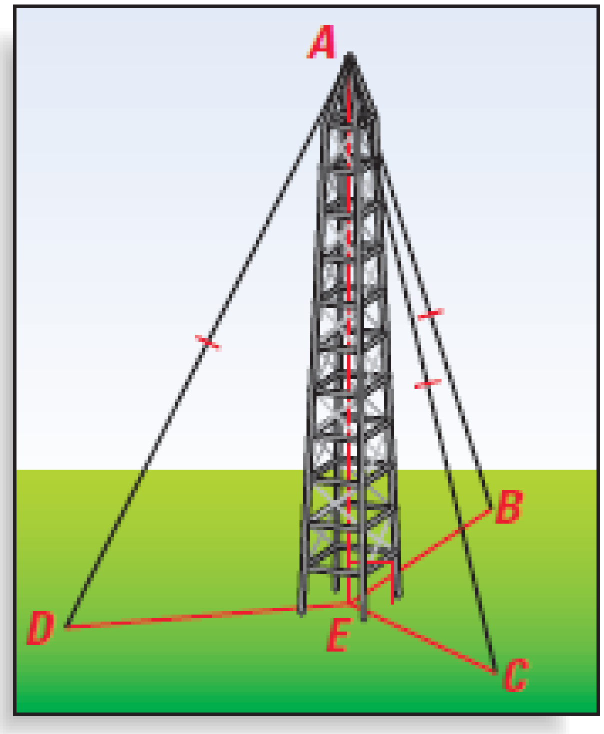
If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.

If $\overline{BC} \cong \overline{EF}$ and $\overline{AC} \cong \overline{DF}$, then $\triangle ABC \cong \triangle DEF$.



EXAMPLE 3 *Proving Right Triangles Congruent*

The television antenna is perpendicular to the plane containing the points B, C, D, and E. Each of the stays running from the top of the antenna to B, C, and D uses the same length of cable. Prove that $\triangle AEB$, $\triangle AEC$, and $\triangle AED$ are congruent.



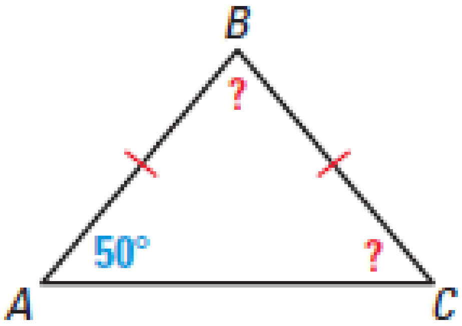
GIVEN ▶ $\overline{AE} \perp \overline{EB}$, $\overline{AE} \perp \overline{EC}$, $\overline{AE} \perp \overline{ED}$, $\overline{AB} \cong \overline{AC} \cong \overline{AD}$

PROVE ▶ $\triangle AEB \cong \triangle AEC \cong \triangle AED$

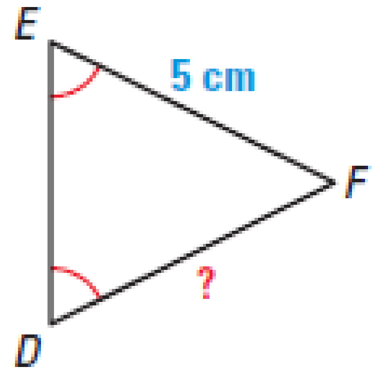
1. Describe the meaning of *equilateral* and *equiangular*.

Find the unknown measure(s). Tell what theorems you used.

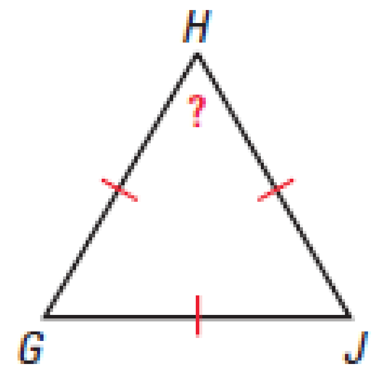
2.



3.

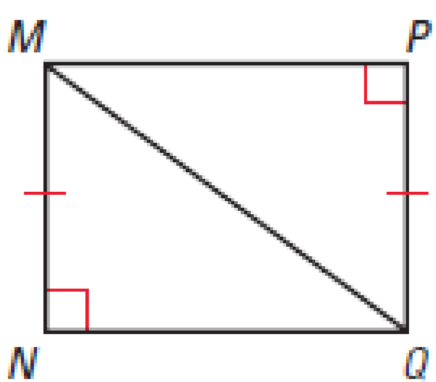


4.

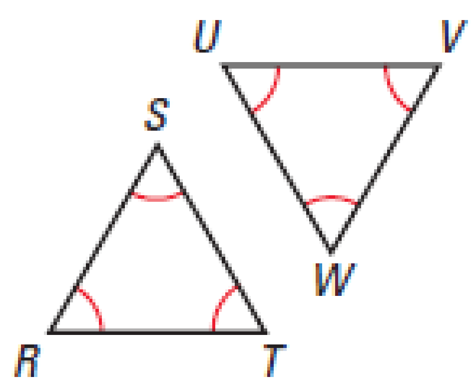


Determine whether you are given enough information to prove that the triangles are congruent. Explain your answer.

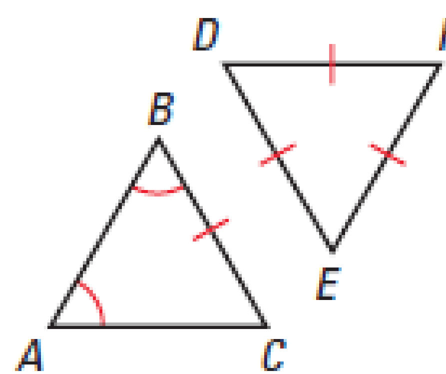
5.



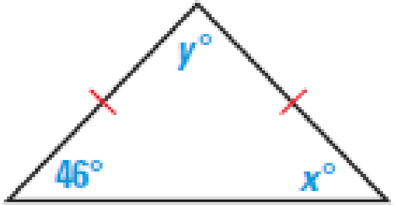
6.

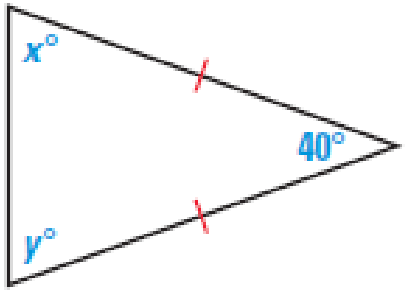


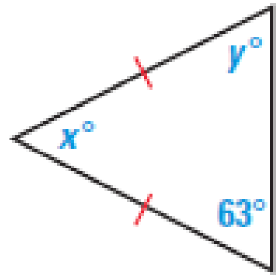
7.



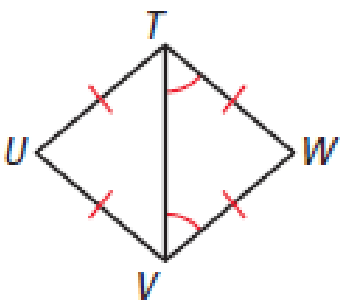
USING ALGEBRA Solve for x and y.

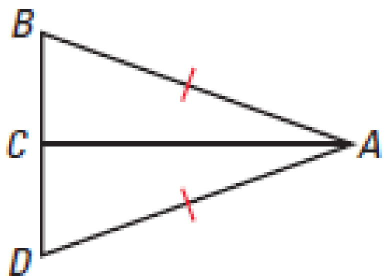
8. 

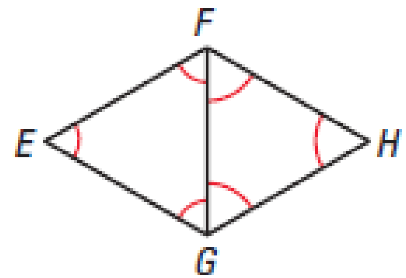
9. 

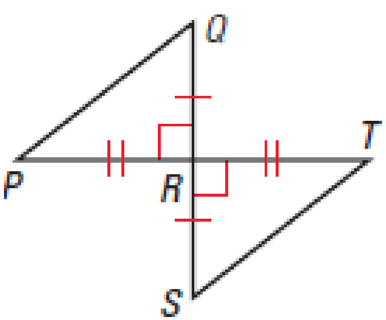
10. 

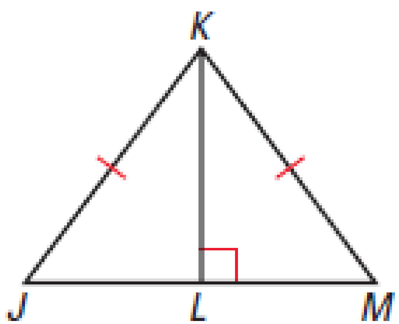
LOGICAL REASONING Decide whether enough information is given to prove that the triangles are congruent. Explain your answer.

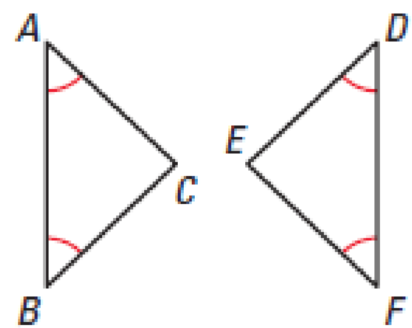
11. 

12. 

13. 

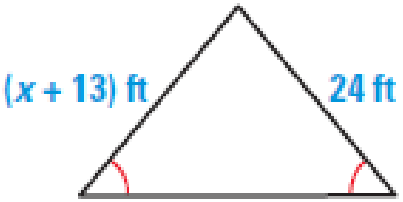
14. 

15. 

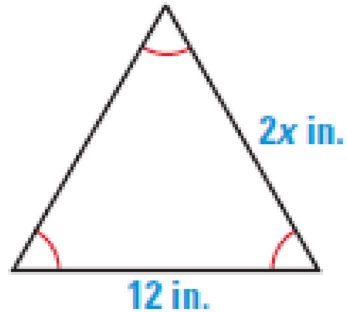
16. 

USING ALGEBRA Find the value of x.

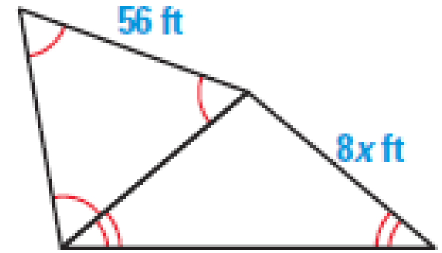
17.



18.

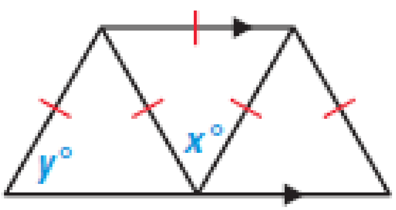


19.

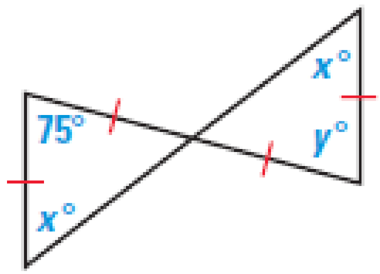


USING ALGEBRA Find the values of x and y.

20.



21.



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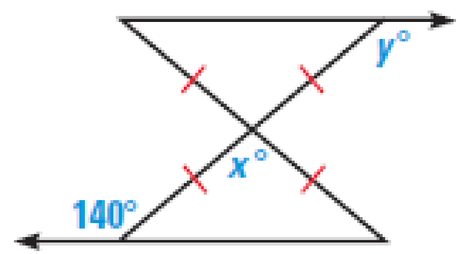
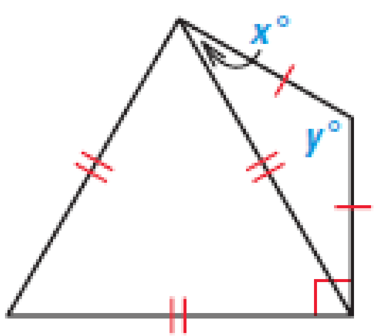
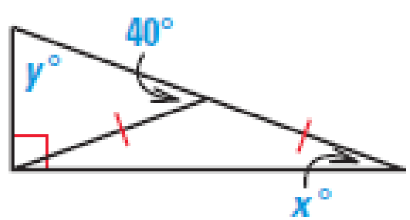


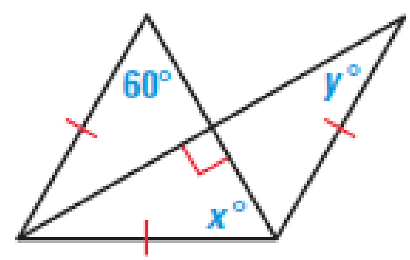
23.



24.



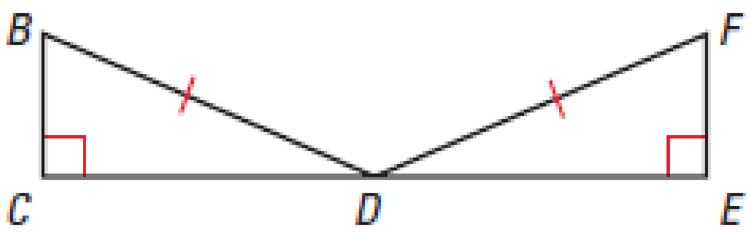
25.



PROOF Write a two-column proof or a paragraph proof.

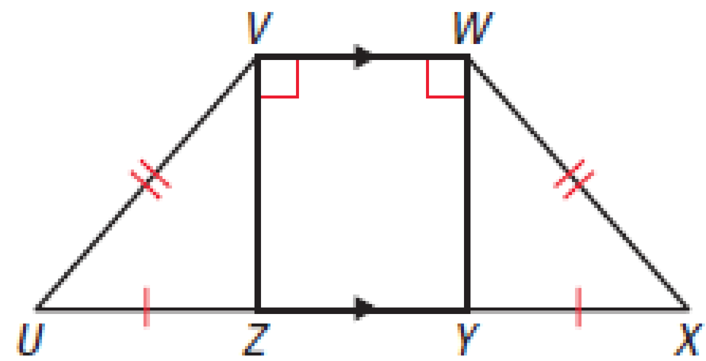
33. GIVEN ▶ D is the midpoint of $\overline{CE}$, $\angle BCD$ and $\angle FED$ are right angles, and $\overline{BD} \cong \overline{FD}$.

PROVE ▶ $\triangle BCD \cong \triangle FED$



34. GIVEN ▶ $\overline{VW} \parallel \overline{ZY}$, $\overline{UV} \cong \overline{XW}$, $\overline{UZ} \cong \overline{XY}$, $\overline{VW} \perp \overline{VZ}$, $\overline{VW} \perp \overline{WY}$

PROVE ▶ $\angle U \cong \angle X$



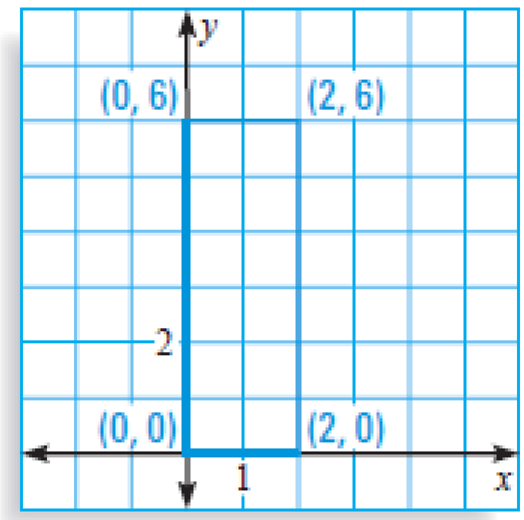
4.7 Triangles and Coordinate Proof

EXAMPLE 1 *Placing a Rectangle in a Coordinate Plane*

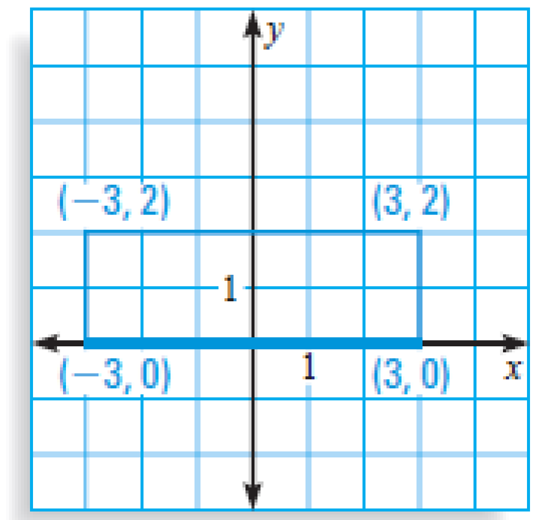
Place a 2-unit by 6-unit rectangle in a coordinate plane.

SOLUTION

Choose a placement that makes finding distances easy. Here are two possible placements.



One vertex is at the origin, and three of the vertices have at least one coordinate that is 0.



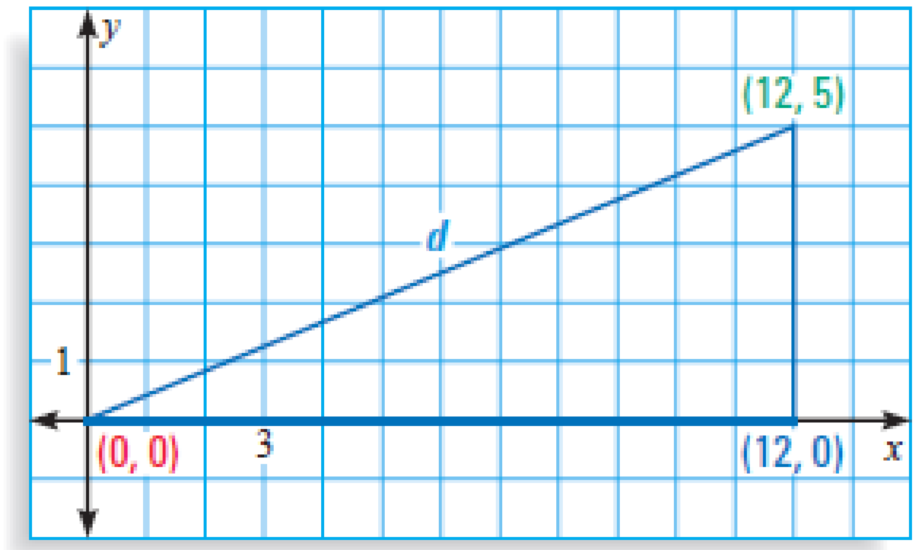
One side is centered at the origin, and the *x*-coordinates are opposites.

EXAMPLE 2 Using the Distance Formula

A right triangle has legs of 5 units and 12 units. Place the triangle in a coordinate plane. Label the coordinates of the vertices and find the length of the hypotenuse.

SOLUTION

One possible placement is shown. Notice that one leg is vertical and the other leg is horizontal, which assures that the legs meet at right angles. Points on the same vertical segment have the same x-coordinate, and points on the same horizontal segment have the same y-coordinate.



You can use the Distance Formula to find the length of the hypotenuse.

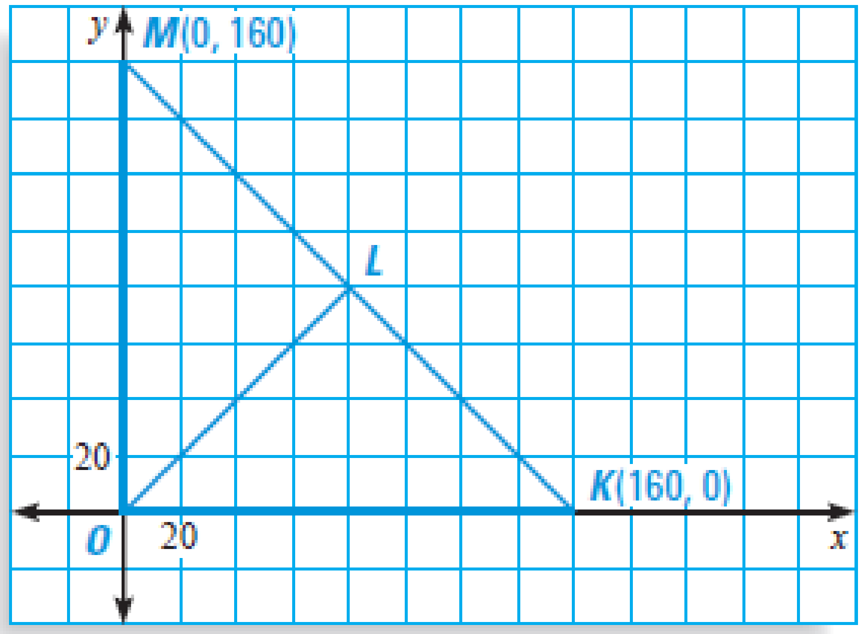
EXAMPLE 3 Using the Midpoint Formula

In the diagram, $\triangle MLO \cong \triangle KLO$.

Find the coordinates of point L.

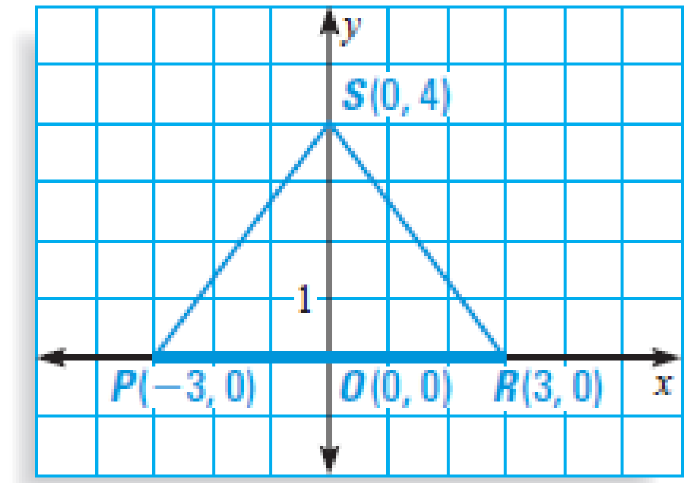
SOLUTION

Because the triangles are congruent, it follows that $\overline{ML} \cong \overline{KL}$. So, point L must be the midpoint of $\overline{MK}$. This means you can use the Midpoint Formula to find the coordinates of point L.



EXAMPLE 4 Writing a Plan for a Coordinate Proof

Write a plan to prove that $\overrightarrow{SO}$ bisects $\angle PSR$.

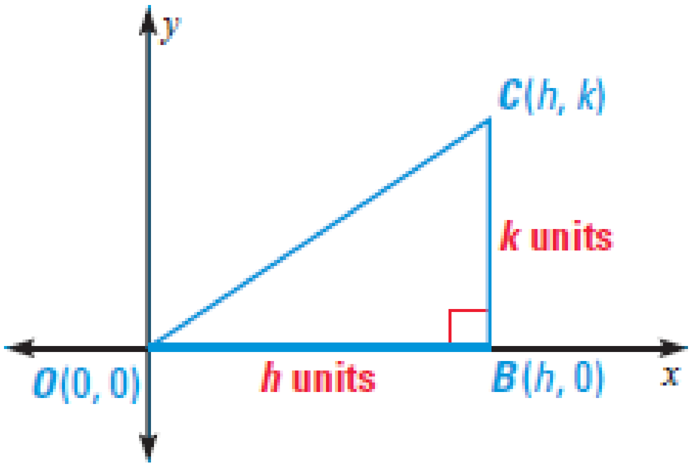


GIVEN ▶ Coordinates of vertices of $\triangle POS$ and $\triangle ROS$

PROVE ▶ $\overrightarrow{SO}$ bisects $\angle PSR$

EXAMPLE 5 Using Variables as Coordinates

Right $\triangle OBC$ has leg lengths of h units and k units. You can find the coordinates of points B and C by considering how the triangle is placed in the coordinate plane.

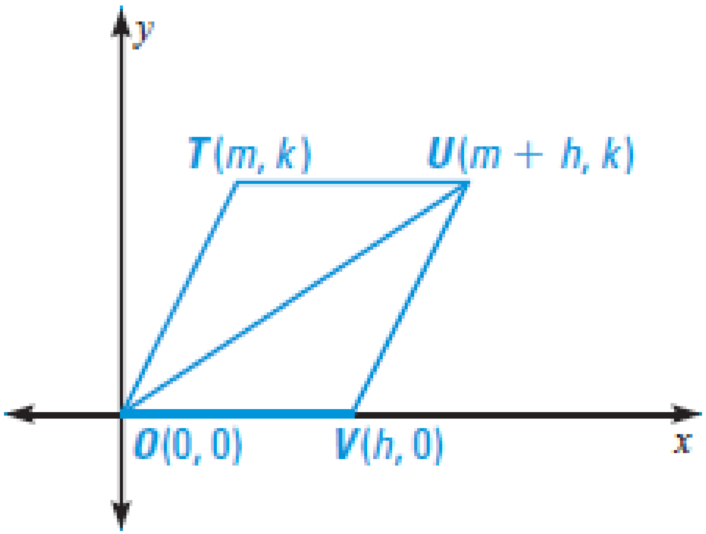


Point B is h units horizontally from the origin, so its coordinates are $(h, 0)$. Point C is h units horizontally from the origin and k units vertically from the origin, so its coordinates are (h, k).

EXAMPLE 6 Writing a Coordinate Proof

GIVEN ▶ Coordinates of figure $OTUV$

PROVE ▶ $\triangle OTU \cong \triangle UVO$



Solution

COORDINATE PROOF Segments $\overline{OV}$ and $\overline{UT}$ have the same length.

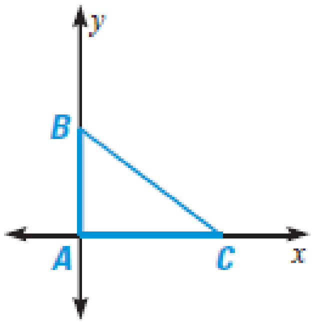
$$OV = \sqrt{(h-0)^2 + (0-0)^2} = h$$

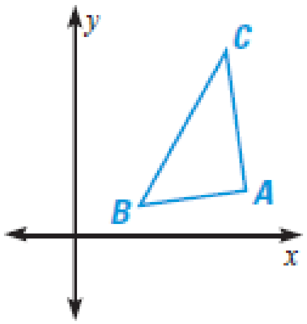
$$UT = \sqrt{(m+h-m)^2 + (k-k)^2} = h$$

Horizontal segments $\overline{UT}$ and $\overline{OV}$ each have a slope of 0, which implies that they are parallel. Segment $\overline{OU}$ intersects $\overline{UT}$ and $\overline{OV}$ to form congruent alternate interior angles $\angle TUO$ and $\angle VOU$. Because $\overline{OU} \cong \overline{OU}$, you can apply the SAS Congruence Postulate to conclude that $\triangle OTU \cong \triangle UVO$.

1. Prior to this section, you have studied two-column proofs, paragraph proofs, and flow proofs. How is a *coordinate proof* different from these other types of proof? How is it the same?

2. Two different ways to place the same right triangle in a coordinate plane are shown. Which placement is more convenient for finding the side lengths? Explain your thinking. Then sketch a third placement that also makes it convenient to find the side lengths.



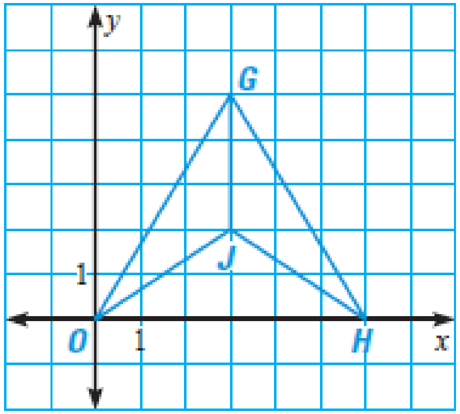


3. A right triangle with legs of 7 units and 4 units has one vertex at (0, 0) and another at (0, 7). Give possible coordinates of the third vertex.

DEVELOPING PROOF **Describe a plan for the proof.**

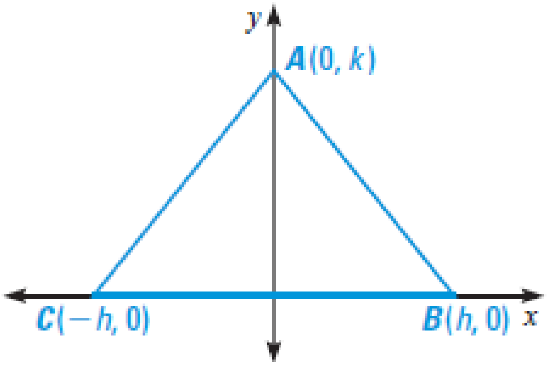
4. **GIVEN** ▶ $\overrightarrow{GJ}$ bisects $\angle OGH$.

PROVE ▶ $\triangle GJO \cong \triangle GJH$



5. **GIVEN** ▶ Coordinates of vertices of $\triangle ABC$

PROVE ▶ $\triangle ABC$ is isosceles.



PLACING FIGURES IN A COORDINATE PLANE Place the figure in a coordinate plane. Label the vertices and give the coordinates of each vertex.

6. A 5-unit by 8-unit rectangle with one vertex at $(0, 0)$

7. An 8-unit by 6-unit rectangle with one vertex at $(0, -4)$

8. A square with side length s and one vertex at $(s, 0)$

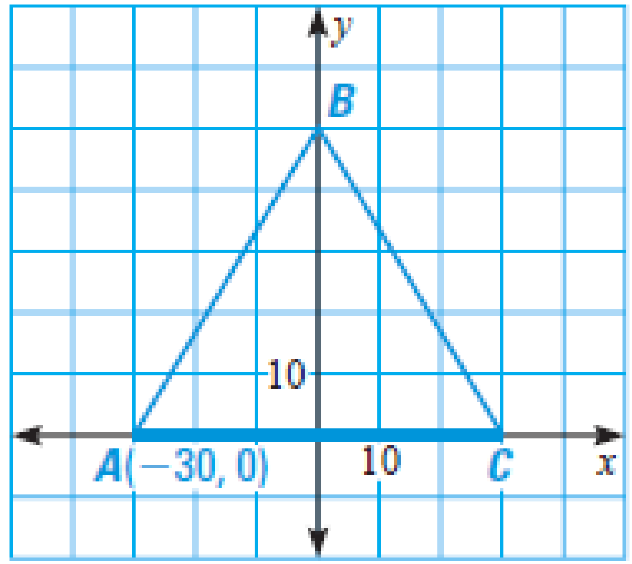
CHOOSING A GOOD PLACEMENT Place the figure in a coordinate plane. Label the vertices and give the coordinates of each vertex. Explain the advantages of your placement.

9. A right triangle with legs of 3 units and 8 units

10. An isosceles right triangle with legs of 20 units

11. A rectangle with length h and width k

FINDING AND USING COORDINATES **In the diagram, $\triangle ABC$ is isosceles. Its base is 60 units and its height is 50 units.**



12. Give the coordinates of points B and C.

13. Find the length of a leg of $\triangle ABC$. Round your answer to the nearest hundredth.

USING THE DISTANCE FORMULA **Place the figure in a coordinate plane and find the given information.**

14. A right triangle with legs of 7 and 9 units; find the length of the hypotenuse.

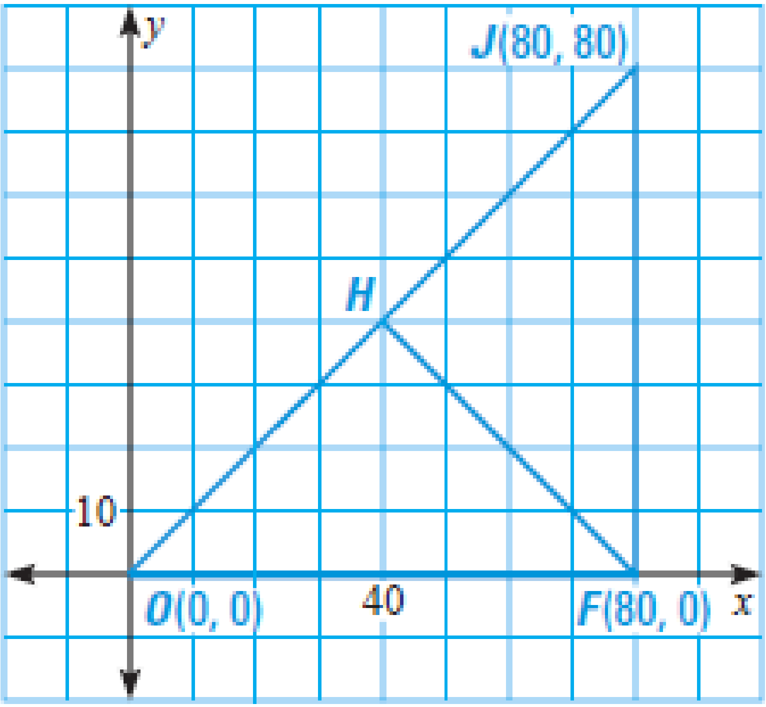
15. A rectangle with length 5 units and width 4 units; find the length of a diagonal.

16. An isosceles right triangle with legs of 3 units; find the length of the hypotenuse.

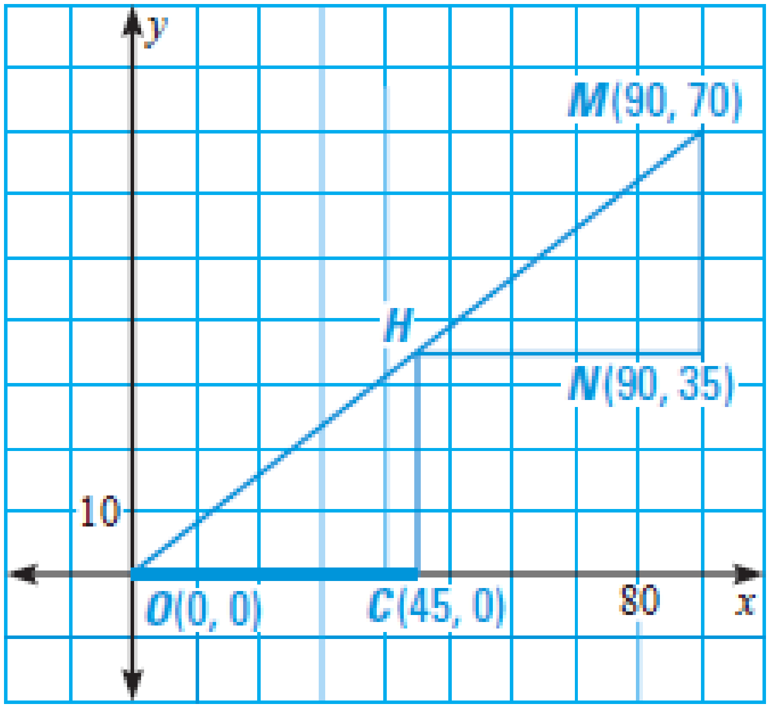
17. A 3-unit by 3-unit square; find the length of a diagonal.

USING THE MIDPOINT FORMULA Use the given information and diagram to find the coordinates of H.

18. $\triangle FOH \cong \triangle FJH$



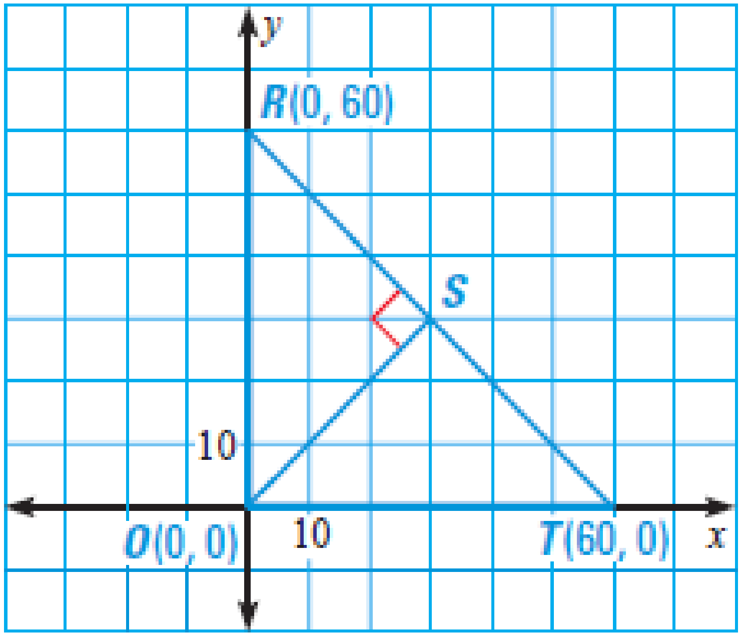
19. $\triangle OCH \cong \triangle HNM$



DEVELOPING PROOF Write a plan for a proof.

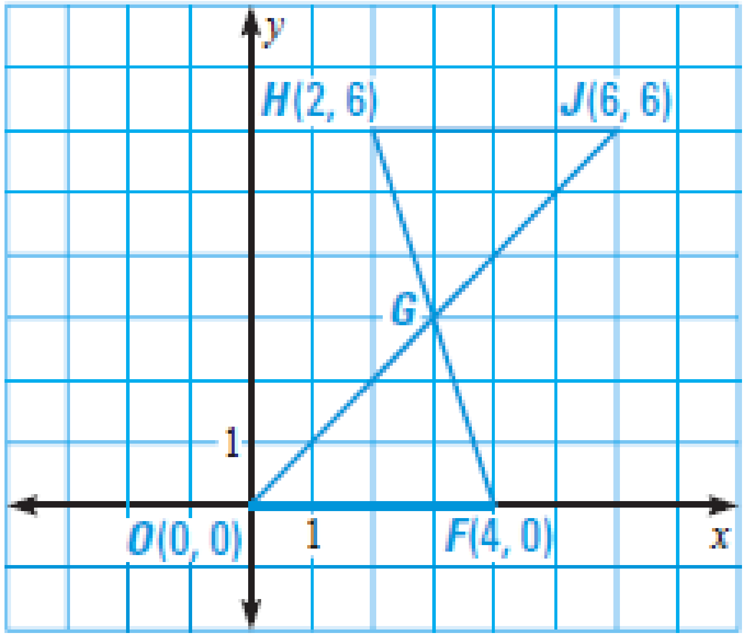
20. **GIVEN** ▶ $\overline{OS} \perp \overline{RT}$

PROVE ▶ $\overrightarrow{OS}$ bisects $\angle TOR$.



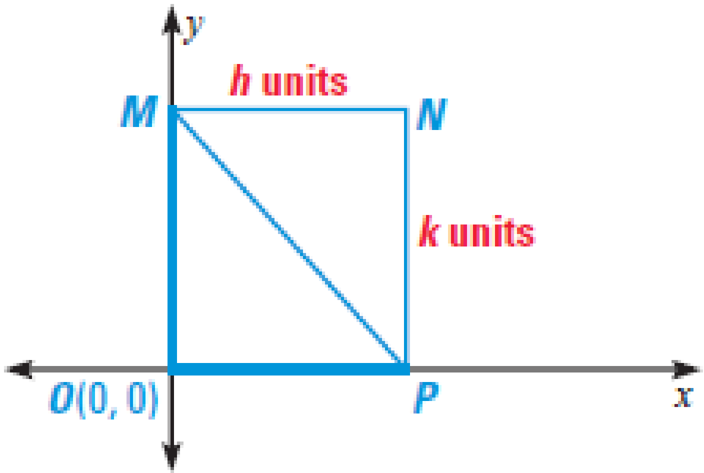
21. **GIVEN** ▶ G is the midpoint of $\overline{HF}$.

PROVE ▶ $\triangle GHJ \cong \triangle GFO$

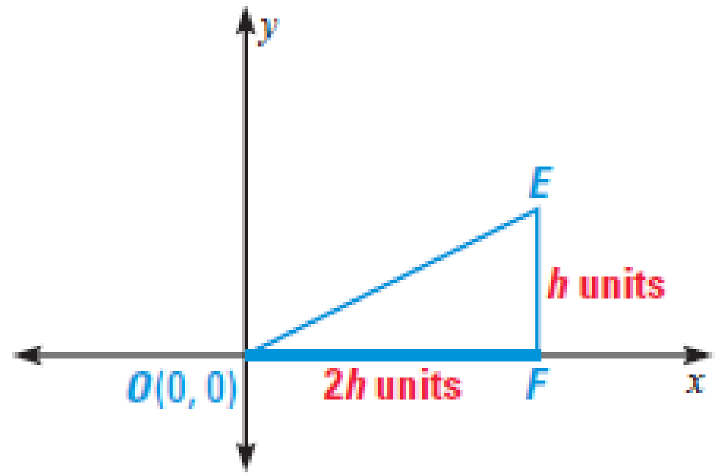


USING VARIABLES AS COORDINATES Find the coordinates of any unlabeled points. Then find the requested information.

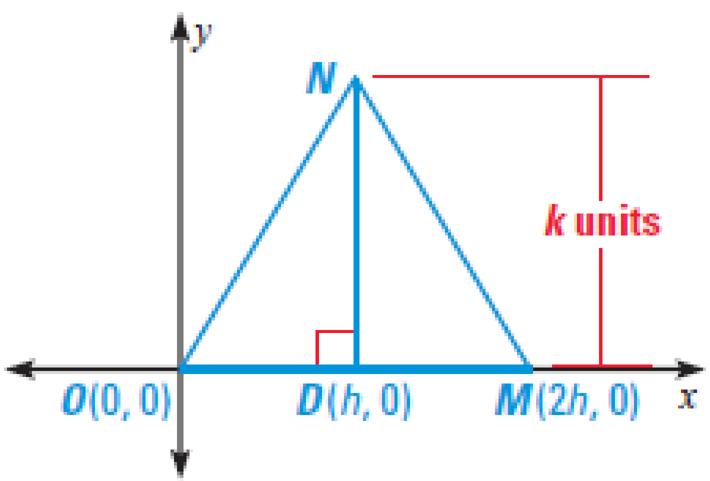
22. Find MP.



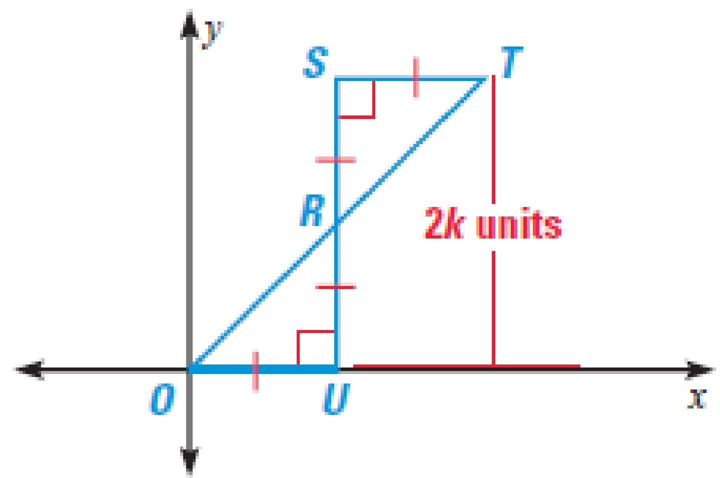
23. Find OE.



24. Find ON and MN.



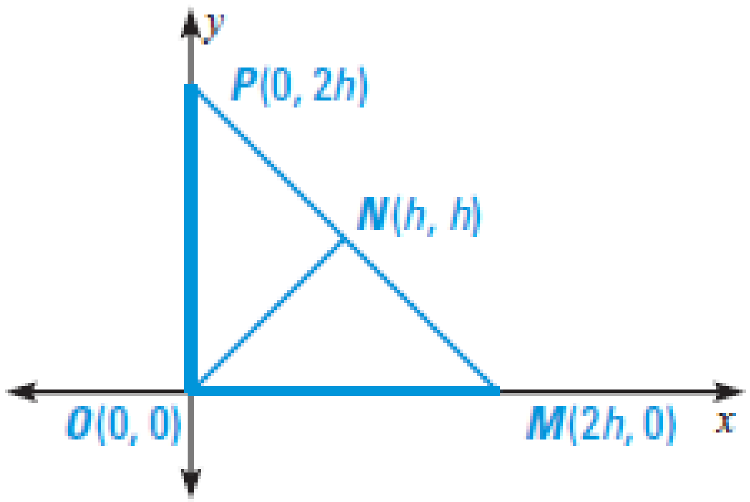
25. Find OT.



COORDINATE PROOF Write a coordinate proof.

26. **GIVEN** ▶ Coordinates of $\triangle NPO$ and $\triangle NMO$

PROVE ▶ $\triangle NPO \cong \triangle NMO$



27. **GIVEN** ▶ Coordinates of $\triangle OBC$ and $\triangle EDC$

PROVE ▶ $\triangle OBC \cong \triangle EDC$