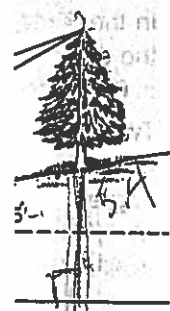


the Statue of Liberty, and
to be $3^\circ 37'$. From the
is $1^\circ 51'$. The distance
meters, and the height
the observer and the
meters.

angle of $8^\circ 15'$ with the
the end of the tree's
ee. Find the height of

the Nile River at a point
the top of the tree from
or lower than the east

of $10^\circ 05'$. The angle
points A and B where
between A and B is
B. Find the height of



Applications: Surveying

Trig



Surveyors often need to obtain a distance that is difficult to measure directly. In that case, the distance is measured indirectly, applying trigonometry to distances and angles that can be measured directly. Since small errors in angle measures may result in large errors in the calculated distance, surveyors use a very sensitive angle measuring instrument called a **transit**.

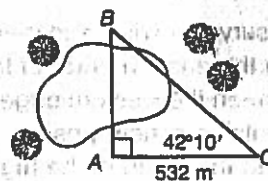
Example

A bridge is to be constructed across a lake and thus the distance between points A and B must be determined. A surveyor has made the measurements shown in the figure. Find the distance between points A and B to the nearest meter.

Solution:

$$\tan 42^\circ 10' = \frac{AB}{532}$$

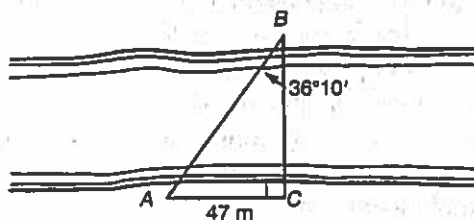
$$\begin{aligned} \text{Therefore, } AB &= 532 \cdot \tan 42^\circ 10' \\ &= 532(.9057) \\ &= 482 \text{ m.} \end{aligned}$$



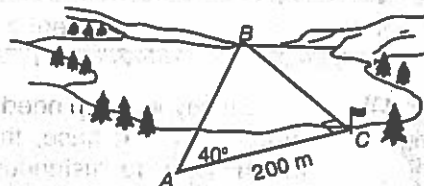
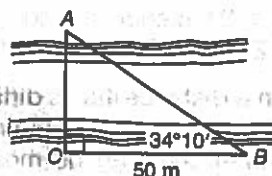
Written Exercises

In Exercises 1–10, unless stated otherwise, compute distances to the nearest meter and angles to the nearest ten minutes.

1. In order to estimate the width of a straight river, a surveyor determined the measurements shown in the figure below. Find the width of the river.

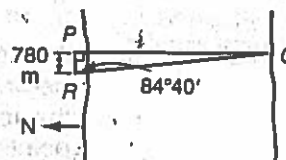


2. To find the distance CA across the river, as shown at the left below, a length CB of 50 meters was measured on one bank. The measure of $\angle B$ was found to be $34^\circ 10'$. Angle C is a right angle. How long is CA ?



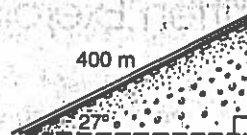
3. To find the distance BC between a flagpole on the shore of Silver Lake and a point on the opposite shore, a surveyor determined the measurements shown in the figure at the right above. Find the distance BC .

4. Points P and Q are on the north and south rims, respectively, of the Grand Canyon, with Q directly south of P . Point R is located 780 meters west of P . The measure of $\angle PRQ$ is $84^\circ 40'$. (See the figure at the right.) Find PQ , the width of the canyon.



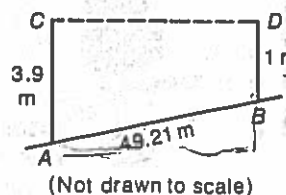
5. A surveyor wishes to determine the width of a north-south highway. While standing on the eastern edge of the highway, he notices a tree at point B , directly opposite him on the western edge. He walks 65 meters from his original position, C , directly south to a new position, A , and finds that the measure of angle BAC is $28^\circ 40'$. Find the width of the highway.
6. A surveyor standing in a gully, finds that the angle of elevation of the top of one side of the gully is 15° . Her eyes are 1.6 meters above the ground and she is standing 4.2 meters from the base of this side. How deep is the gully?

7. The **angle of inclination** of a highway or railroad is the angle formed by the roadbed and the horizontal. At the steepest place, the angle of inclination of the railroad that runs to the summit of Pikes Peak is 27° . How many meters would you rise vertically in traveling 400 meters along this track?



b

8. In traveling along a highway or railroad built on a slope, your position changes vertically as well as horizontally. The ratio of vertical distance to horizontal distance is called the **grade**. To find the grade of a proposed highway, a surveyor places a leveling instrument on the slope AB , and the line CD is sighted to two upright rods. If $AC = 3.9$ meters, $BD = 1$ meter, and $AB = 49.21$ meters, find the grade and the angle of inclination.



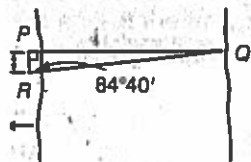
9. To find the distance CE , a surveyor measured the distance AC and the angle $\angle C$.

10. An observer standing on a hill passes a point on the hill and starts walking down the hill.

left below, a length
of $\angle B$ was found to

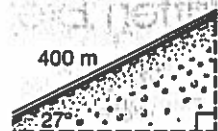


f Silver Lake and a
surements shown in

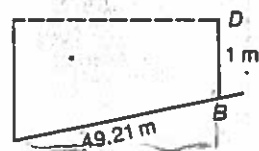


way. While standing
B, directly opposite
position, C, directly
angle BAC is $28^\circ 40'$.

ion/of the top of one
ground and she is
the gully?

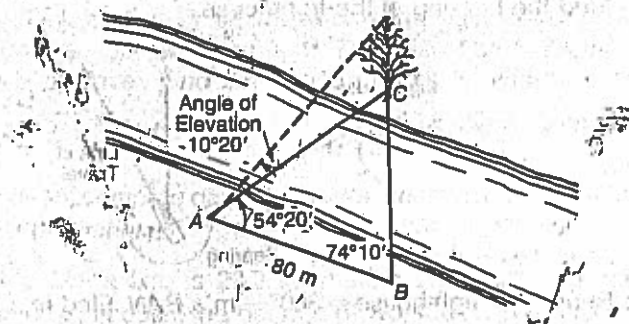


our position changes



(Not drawn to scale)

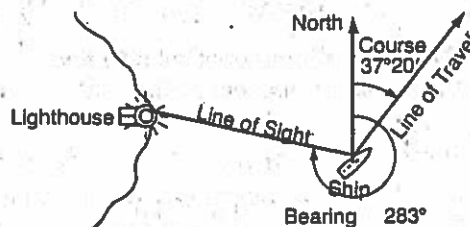
9. To find the height of a tree standing at point C across a river from point A, a base line, AB, 80 meters long is established on one side of the river. The measure of $\angle CBA$ was found to be $74^\circ 10'$. The angle of elevation of the top of the tree from A measures $10^\circ 20'$. Find the height of the tree.



10. An exit ramp with a .0496 grade is to be constructed from Interstate 201 to an overpass 10.7 meters above the horizontal level of the road. A surveyor must locate the position on the road at which the ramp will start. Find the distance measured along the horizontal from the position of the overpass to the position at which the ramp will start. Write your answer to the nearest tenth of a meter.

Applications: Navigation

In navigation, the **course** of a ship or plane is the angle measured clockwise from north to the line of travel. Thus, in the figure below, the course of the ship is $37^\circ 20'$.



The **bearing** of a line of sight is the angle measured clockwise from north to the line of sight. Thus, the bearing of the lighthouse in the figure above is 283° .

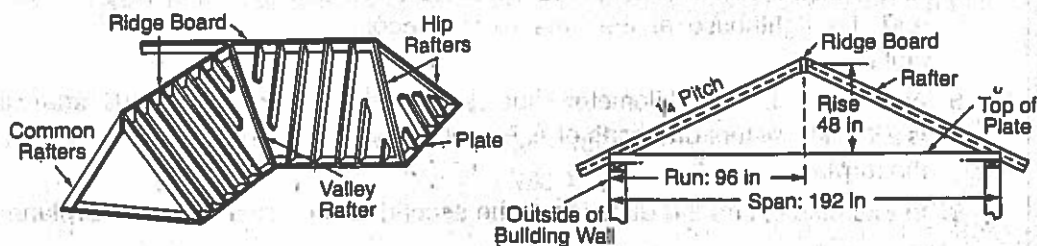
- b
13. At 6 A.M., a ship is sailing due south at a constant speed. The navigator sights a lighthouse on a bearing of 270° at a distance of 24.2 kilometers. At 6:30 A.M., the bearing of the lighthouse is 285° . Find, to the nearest kilometer per hour, the rate at which the ship is sailing.
 14. The ship of Exercise 13 continues sailing south at a constant rate. Find the bearing of the lighthouse at 9:00 A.M.
 15. Two ships leave the same harbor at the same time. One sails at a constant rate of 40 kilometers per hour on course $42^\circ 15'$, and the other sails at a constant rate of 51 kilometers per hour on course $132^\circ 15'$. How far apart are the ships three hours after leaving the harbor?

Example

Solut

1-12 Applications: Construction

The figure at the left below shows the names of some of the structural members in the frame of a roof.



The **span** of a roof is the distance between the outside walls. For a symmetrical roof, the **run** is one half the span. The **rise** of a roof is the distance between the center of the ridge board and the level of the top of the plate. Architects and carpenters refer to the ratio

$$\frac{\text{Rise}}{\text{Span}}$$

as the **pitch** of the roof. In the figure above, a roof is shown with a pitch of $\frac{1}{4}$.

Written

1. Rise
3. Rise

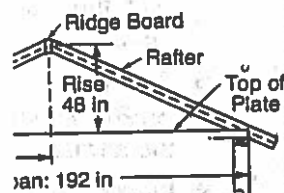
5. Spa
7. Run

re navigator sights a
ters. At 6:30 A.M., the
ter per hour, the rate

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s at a constant rate of
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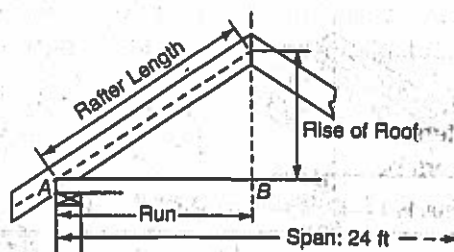


e outside walls. For a
he **rise** of a roof is the
and the level of the top
e ratio

of is shown with a pitch

Example

Find, to the nearest inch, the rafter length for a symmetrical roof with a $\frac{1}{3}$ pitch and a span of 24 feet.



Solution: First, find the rise and run.

$$\begin{aligned}\text{Run} &= \frac{1}{2} \times \text{Span} \\ &= \frac{1}{2} \times 288 = 144 \text{ inches} \quad \leftarrow 24 \text{ feet} = 288 \text{ inches}\end{aligned}$$

$$\text{Since Pitch} = \frac{\text{Rise}}{\text{Span}},$$

$$\begin{aligned}\text{Rise} &= \text{Pitch} \times \text{Span} \\ &= \frac{1}{3} \times 288 = 96 \text{ Inches}\end{aligned}$$

$$\begin{aligned}\text{Then rafter length} &= \sqrt{144^2 + 96^2} \\ &= \sqrt{29,952}, \text{ or about } 173 \text{ inches}\end{aligned}$$

Written Exercises

In Exercises 1–4, find the pitch of a symmetrical roof for the given dimensions.

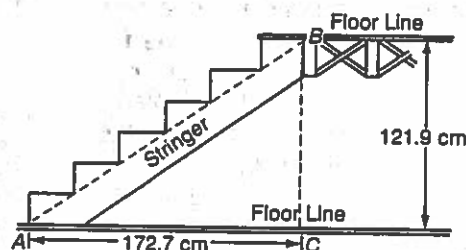
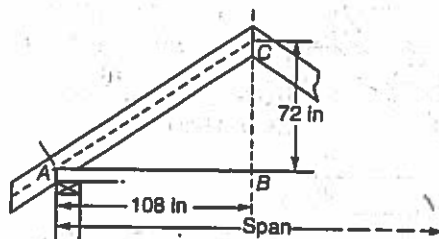
1. Rise: 83 in; Span: 249 in
2. Rise: $4\frac{1}{2}$ ft; Span: 6 ft
3. Rise: 200 cm; Run: 400 cm
4. Rise: 1.8 m; Run: 1.2 m

In Exercises 5–8, find the rise of each roof. Assume it is symmetrical.

5. Span: 488 cm; Pitch: $\frac{1}{4}$
6. Span: 2.2 m; Pitch: $\frac{1}{2}$
7. Run: 111 in; Pitch: $\frac{1}{3}$
8. Run: 3 yd; Pitch: $\frac{2}{3}$

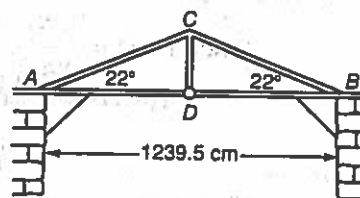
In Exercises 9–14, compute lengths to the nearest unit and angles to the nearest ten minutes.

9. Find the pitch of the symmetrical roof shown in the figure at the left below.

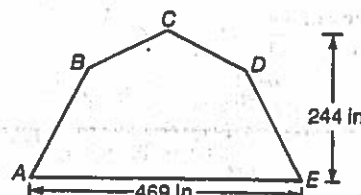
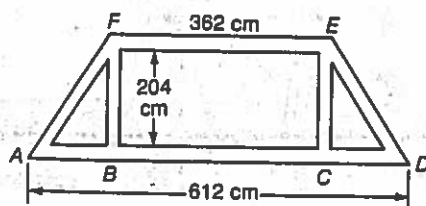


10. A piece of lumber cut to support the treads of a stair as shown in the figure at the right above is called a **stringer**. Find the length, AB , of the piece of lumber required for the stringer.

11. A truss for a bridge is to be constructed with measurements as shown in the figure at the right. Find the length, CD , of the vertical tie rod.



12. A steel bridge has a truss with measurements as shown in the figure at the left below. Find the measure of angle FAD .



13. In the gambrel roof shown in the figure at the right above, the upper rafters have a pitch of $\frac{1}{4}$ and the lower rafters have a pitch of 1. Find the lengths AB and CD .

14. Two rafters are sometimes joined by a **collar beam** for extra strength. Find the length, CD , of the collar beam in the figure at the right if the roof has a pitch of $\frac{1}{3}$.

